Water distribution network optimisation using a modified central force optimisation method

Azadeh Jabbary PhD
PhD Graduate, Department of Agricultural Engineering, Lorestan University, Khoramabad, Iran; member of Young Researchers and Elite Club, Science and Research Branch, Islamic Azad University, Tehran, Iran

Hasan Torabi Podeh PhD
Assistant Professor, Department of Agricultural Engineering, Lorestan University, Khoramabad, Iran (corresponding author: torabi.ha@lu.ac.ir)

Hojatollah Younesi PhD
Assistant Professor, Department of Agricultural Engineering, Lorestan University, Khoramabad, Iran

Amir Hamzeh Haghiabi PhD
Associate Professor, Department of Agricultural Engineering, Lorestan University, Khoramabad, Iran

In the present research, the application of a proposed, modified, central force optimisation method for multi-objective optimisation of water distribution systems is examined. This new method is based on a central force optimisation algorithm in which the non-dominated sorting plus crowding distance calculator mechanisms produce optimal Pareto solutions in objective functions trade-offs. Minimising pipe-sizing cost and maximising new reliability measures are considered as objective functions. The method is written in Matlab and linked with Epanet water distribution software. Employing the method to solve the Kadu and Khorramshahr water distribution networks shows its capability to produce well-distributed optimal Pareto solutions for cost and reliability. Also, by utilising a fuzzy ranking method, a best compromise solution for two objectives can be extracted from the optimal Pareto set.

**Notation**

- \( \mathbf{d}_j \): probe \( p \)'s acceleration at step \( j - 1 \)
- \( C_j \): uniformity of node \( j \)
- \( C_i \): total cost of network
- \( c(D_i, L_i) \): cost of pipe \( i \) with diameter \( D_i \) and length \( L_i \)
- \( D_i \): diameter of pipe \( i \)
- \( D_{\text{max}} \) and \( D_{\text{min}} \): maximum and minimum commercial diameters, respectively
- \( F_k^j \): amount of \( f_{\text{th}} \) objective function for solution \( k \)
- \( F_{\text{max}}^j \) and \( F_{\text{min}}^j \): maximum and minimum amounts of \( f_{\text{th}} \) function among optimal Pareto solutions, respectively
- \( F_{\text{rep}} \): repositioning factor
- \( f_i \): feasibility index
- \( H_j \) and \( H_j^f \): head and allowable minimum head at node \( j \), respectively
- \( I_k \): failure index
- \( I_n \): network resilience index
- \( I_{\text{ef}} \): feasible network resilience index
- \( I_e \): resilience index
- \( L_i \): length of pipe \( i \)
- \( M \): number of non-dominated solutions
- \( M_{p}^j \): probe \( p \)'s fitness at step \( j - 1 \)
- \( N_j \): total number of problem’s dimensions
- \( N_i \): total number of pipes in network
- \( N_j \): total number of nodes in network
- \( N_{\text{obj}} \): number of objective functions
- \( N_p \): total number of probes
- \( N_{\text{ps}} \): total number of network’s pumps
- \( N_r \): total number of reservoirs in system
- \( N_i \): maximum number of iterations
- \( N_{pj} \): total number of pipes connected to node \( j \)
- \( P_j \) and \( Q_j \): pressure and demand in node \( j \), respectively
- \( P_{\text{max}} \) and \( P_{\text{min}} \): allowable maximum and minimum pressure point in the nodes, respectively
- \( P_{\text{ps}} \): total power entered into the network by each pump
- \( Q_r \) and \( H_r \): total discharge and total head relevant to reservoir \( r \), respectively
- \( R_{j}^{m} \): position of probe \( k \) in the \( m \)th dimension at step \( j - 1 \)
- \( U(.) \): unit step function
1. Introduction

In the past few decades, many researchers have focused on finding the optimal cost of pipe sizing of water distribution networks (WDNs) by utilising meta-heuristic optimisation algorithms (Haghighi et al., 2011; Kadu et al., 2008; Mohammadi-Aghdam et al., 2015). They have shown that these methods were successful in solving a single objective WDN problem. However, a WDN design with the least pipe-sizing cost could not have the sufficient surplus energy that is required to meet future demands or to overcome the failure conditions during the operational period of the WDN (Gessler and Walski, 1985; Prasad et al., 2003; Todini, 2000). The research relevant to this issue emphasised the need to consider the WDN as a multi-objective problem, minimising the cost function and maximising the benefit function of the WDN. So far, various contractual indices have been defined for the benefit function. Gessler and Walski (1985) considered the benefit as the amount of excess head over minimum allowable pressure at the worst node of the network.

Xu and Goulter (1999) defined the concept of network reliability as the ability of a WDN to show appropriate performance during normal or even abnormal operational periods. Todini (2000) was the first to define the resilience as the capability of the network to overcome failure conditions. Todini (2000) considered the resilience index \( I_r \) as a reliability measure and obtained the optimal Pareto-front solutions of the network in the cost–reliability space. Also Todini (2000) defined the failure index \( I_f \) to evaluate the pipe failure effect in the network.

In general, failure conditions of the WDN include hydraulic failures (changes in demands or head, inadequate pipe sizing and so on) and mechanical failures (pipe breakage, control valve failure, pumps failure and so on) (Mays, 1996). Prasad et al. (2003) mentioned that maximisation of \( I_r \) and \( I_f \) indices might be able to raise the surplus head of the network, but these indices do not consider the effect of redundancy. They proposed the network resilience index \( I_d \) as a reliability measure by considering the effect of redundancy in the network and solved the WDN problem in cost–\( I_n \) space utilising a non-dominated sorting genetic algorithm (NSGA). In fact, redundancy reflects the reliability of loops in the network, meaning that the loops in the WDN should not meet any big diameter change at the nodes (Prasad et al., 2003).

All in all, the reliability indices \( (I_r, I_f \) and \( I_d) \) represent the network power to compensate the losses in failure conditions during the operational period. In particular, \( I_n \) reflects the existing surplus head and the applicability of the loops in a WDN design; but this index is unable to represent the feasibility or the infeasibility status of the solutions. Thus, in the current research a feasibility index has been applied on the network resilience index. The new index, termed the feasible network resilience \( I_{fd} \), is utilised as a second objective function in the multi-objective WDN optimisation.

From the literature review of utilising stochastic meta-heuristic algorithms in WDN design, it can be admitted that these methods are successful in solving the WDN problem. However, owing to the stochastic nature of these algorithms, there is no guarantee that the global optimal solution will be obtained in a WDN single-objective problem. Accordingly, the generated non-dominated solutions in each run of the multi-objective WDN problem, utilising these stochastic algorithms, are not always the same. So, several runs are essential to assure the quality of the incidental best solution. Therefore, the computational time to find a near-optimal configuration of a real WDN is a limiting issue in utilising these stochastic algorithms.

Owing to these drawbacks of stochastic methods and, as the problem of the WDN has never been solved by any completely deterministic meta-heuristic method, utilisation of the central force optimisation (CFO) method has been investigated in this research. CFO (Formato, 2007) is a multi-dimensional method which performs based on the physical gravitational law. All equations in CFO are inherently deterministic. Therefore, the results of every CFO run with a similar set-up will be the same (Formato, 2007). This method has been successfully applied to different problems, such as microwave broadband absorber design (Asi and Dib, 2010) and antenna optimisation (Formato, 2010). A high efficiency of this optimisation method has been shown in these research studies. Moreover, in some of them, the researchers have increased CFO performance by applying some modifications to the algorithm.

In the present research, a modified CFO method is introduced for multi-objective optimisation of WDNs (MCFOnet). For this purpose, MCFOnet is linked with Epanet 2.0 hydraulic software (Rossman, 2000) in Matlab. The capital cost and the feasible network resilience index have been considered as the

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V_i \quad \text{velocity in pipe } i
\]

\[\nu(PV) \quad \text{total penalty value associated with pressure and velocity constraints}\]

\[\nu(V_i) \text{ and } \nu(P_i) \quad \text{penalty functions of velocity and pressure constraints, respectively}\]

\[x_i \quad \text{variable in central force optimisation}\]

\[x_i^{\text{max}} \text{ and } x_i^{\text{min}} \quad \text{maximum and minimum decision variables, respectively}\]

\[\alpha, \beta, G \quad \text{central force optimisation constant parameters}\]

\[\gamma \quad \text{specific weight of water}\]

\[\vartheta \quad \text{penalty function coefficient}\]

\[\mu^k \quad \text{rank of each solution } k\]

\[\mu^f_{I_f} \quad \text{membership function value of each solution } k \text{ for each } f \text{ function}\]