ON NEW CLASSES OF MULTICONE GRAPHS DETERMINED BY THEIR SPECTRUMS

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Abstract. A multicone graph is defined to be join of a clique and a regular graph. A graph $G$ is cospectral with graph $H$ if their adjacency matrices have the same eigenvalues. A graph $G$ is said to be determined by its spectrum or DS for short, if for any graph $H$ with $Spec(G) = Spec(H)$, we conclude that $G$ is isomorphic to $H$. In this paper, we present new classes of multicone graphs that are DS with respect to their spectrums. Also, we show that complement of these graphs are DS with respect to their adjacency spectrums. In addition, we show that graphs cospectral with these graphs are perfect. Finally, we find automorphism group of these graphs and one conjecture for further researches is proposed.

1. Introduction

In this paper, we are concerned only with finite undirected simple graph (loops and multiple edge are not allowed). All terminology and notation on graphs not defined here can be found

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in [2, 5, 9]. Let $\Gamma = (V, E)$ be a simple graph, where $V$ is the set of vertices and $E$ is the set of edges of $\Gamma$. An edge joining the vertices $u$ and $v$ is denoted by $\{u, v\}$. Permutation of a set $\Omega$ is a bijection from $\Omega$ to itself. Sym($\Omega$) denotes the set of all permutations of $\Omega$. $S_n$ denotes the symmetric group $sym\{1, 2, ..., n\}$. Automorphism of a simple graph $\Gamma = (V, E)$ is a permutation of the vertices of $\Gamma$ which preserves the relation of adjacency; that is, a bijection $\pi : V \rightarrow V$ such that $\{u, v\} \in E$ if and only if $\{\pi(u), \pi(v)\} \in E$. Under composition, the automorphisms form a group, called the automorphism group (or symmetry group) of $\Gamma$, and this is denoted by $Aut(\Gamma)$. In most situations it is very difficult to determine the automorphism group of a graph and this has been the subject of many research papers. Some of recent works appear in references [7, 8, 9]. The complement of a graph $G$, denoted by $\overline{G}$, is the graph on the vertex set of $G$ such that two vertex of $\overline{G}$, are adjacent if an only if they are not adjacent in $G$. A graph and its complement have the same automorphisms. The automorphism group of the complete graph $K_n$ and the empty graph $\overline{K_n}$ is the symmetric group $S_n$. The union of two vertex disjoint graphs $G_1$ and $G_2$ denoted by $G_1 \cup G_2$, is the graph whose vertex (respectively, edge) set is the union of vertex (respectively, edge) sets of $G_1$ and $G_2$. The join of two vertex disjoint graphs $G_1$ and $G_2$ is the graph obtained from $G_1 \cup G_2$ by joining each vertex in $G_1$ with every vertex in $G_2$. It is denoted by $G_1 \vee G_2$. The group $G$ is called a semidirect product of $N$ by $Q$, denoted by $G = N \rtimes Q$, if $G$ contains subgroups $N$ and $Q$ such that (i) $N \subseteq Q(N$ is a subgroup normal of $G)$; (ii) $NQ = G$; (iii) $N \cap Q = 1$. If $\Gamma$ and $\Delta$ are nonempty sets then we write $Fun(\Gamma, \Delta)$ to denote the set of all functions $\Gamma$ into $\Delta$. In the case when $K$ is a group, we can turn $Fun(\Gamma, \Delta)$ into a group by defining a product “pointwise”: $fg(\gamma) = f(\gamma)g(\gamma)$, for all $f, g \in Fun(\Gamma, \Delta)$ and $\gamma \in \Gamma$. and the product on the right is in $K$. In the case that $\Gamma$ is finite of size $m$, say $\Gamma = \{\gamma_1, \gamma_2, ..., \gamma_m\}$ then the group $Fun(\Gamma, \Delta)$ is isomorphic to $K^m$ (a direct product of $m$ copies of $K$) via the isomorphism. Let $K$ and $H$ be groups and suppose $H$ acts on the nonempty set $\Gamma$. Then the wreath product of $K$ by $H$ with respect to this action is defined to be the semidirect $Fun(\Gamma, K) \rtimes H$ where $H$ acts on the group $Fun(\Gamma, \Delta)$ via $f^x(\gamma) = f(\gamma^x)$ for all $f \in Fun(\Gamma, \Delta)$, $\gamma \in K$ and $x \in H$. We denote this group by $K \ wr H$.

Let $A(G)$ denotes the $(0, 1)$-adjacency matrix of graph $G$. The characteristic polynomial of $G$ is $\det(\lambda I - A(G))$, and is denoted by $P_G(\lambda)$. The roots of $P_G(\lambda)$ are called the adjacency eigenvalues of $G$ and since $A(G)$ is real and symmetric, the eigenvalues are real numbers. If $G$ has $n$ vertices, then it has $n$ eigenvalues in descending order as $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_n$. Let $\lambda_1, \lambda_2, ..., \lambda_s$ be the distinct eigenvalues of $G$ with multiplicity $m_1, m_1, ..., m_s$, respectively. The multi-set $Spec(G) = \{\lambda_1^{m_1}, \lambda_2^{m_2}, ..., \lambda_s^{m_s}\}$ of eigenvalues of $A(G)$ is called the adjacency spectrum of $G$. For two graphs $G$ and $H$, if $Spec(G) = Spec(H)$, we say $G$ and $H$ are cospectral with respect to adjacency matrix. A graph $H$ is said to be determined by its spectrum or DS for short, if for a graph $H$ with $Spec(G) = Spec(H)$, one has $G$ isomorphic to $H$. Let $G$