Assessment of effect of SSSC stabilizer in different control channels on damping inter-area oscillations

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A B S T R A C T

A static synchronous series compensator (SSSC) is one of the series flexible ac transmission system (FACTS) devices that injects a balanced three-phase voltage in quadrature with the transmission line current. There are two channels for controlling phase and magnitude of the voltage. When the SSSC is used for damping of inter-area oscillations, a SSSC-based stabilizer can be included in both channels. In this paper, the best location and suitable input control signal for SSSC in order to enhance the damping of inter-area oscillations are selected by residue analysis. A method by quadratic mathematic programming has been presented to the design of the stabilizer. By this method, the effect of the stabilizer in both control channels of the SSSC on damping of inter-area oscillations has been assessed. By considering the gain of stabilizer as a criterion, obtained results from studying on a small and a large multi-machine power system show that the stabilizer in the phase control channel is more effective for damping inter-area oscillations.

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1. Introduction

Inter-area oscillations, ranging 0.1–0.7 Hz, are common phenomena in the interconnected power systems. To provide a secure operation for power plants and energy saving, damping of these oscillations have become one of the major problems in the power system stability and have received a great deal of attentions [1]. If not well damped, magnitude of these oscillations may keep growing until loss of synchronism results. Conventionally, power system stabilizers (PSSs) are used for damping of power system oscillations. However, the use of PSSs only may not be, in some cases, effective in providing sufficient damping for inter-area oscillations, particularly with increasing transmission line loading over a long distance [2]. Also, PSSs suffer a drawback of being liable to cause great variations in the voltage profile and they may even result in leading power factor operation [3]. Nowadays, series power electronics-based FACTS controllers have become one of the best alternative means to improve power oscillation damping. The SSSC is one of series FACTS devices that has been initiated by Gyugyi et al. [4]. In addition to increasing transferred power, it can improve transient and small-disturbance stability of power systems [5,6]. The SSSC, in comparison to other FACTS devices, is more effective for damping mechanical oscillations [7,8]. Of course, location and input signal has important effect on its performance [9]. The SSSC injects a set of balanced voltages to transmission line quadrature with the line current. There are two control channels to control the magnitude and phase of the voltage, which are the magnitude control channel and the phase control channel. When the SSSC is used for damping of mechanical oscillations, the damping stabilizer can be included in both channels. Mechanical oscillations in a single machine infinite bus power system can be damped by a SSSC-based stabilizer in the magnitude control channel [10] or by a SSSC stabilizer in the phase control channel [11]. SSSC-based stabilizer in the magnitude control channel also can be used for damping mechanical oscillations in a multi-machine power system [12]. But, it seems that the effects of SSSC stabilizer in the phase control channel on damping inter-area oscillations for a multi-machine power system has not been studied in the reported literature. Also, in the pertinent literature any comparative study between effect of SSSC stabilizer in the different control channels on damping mechanical oscillations, particularly, inter-area oscillations have been presented.

One of the effective methods to design damping stabilizers in FACTS devices is linear programming [13,14], but in these papers the first phase of controller has been calculated and assuming that stabilizer phase remains constant in the frequency range of oscillation modes; the gain of the stabilizer, using the linear programming, is calculated. This assumption may be not true. Therefore in [15,16] gain and phase PSS, using quadratic mathematic programming (QMP), have simultaneously been calculated. The PSSs are located on the generators to damp out local oscillations but series FACTS devices are installed on transmission lines to improve damping inter-area oscillations. Investigation on using QMP method for
designing FACTS-based stabilizers in the multi-machine power systems can be useful for power system designers. On the other hand, in [15,16], it is supposed that the phase of the residue related to a critical mode is positive; consequently the PSS must have a lead structure. However, in some cases, particularly in the FACTS-based stabilizers the phase of residue can be negative and the stabilizer must have a phase-lag structure.

In this paper, the most effective location and input control signal for SSSC are selected by residue analysis. Design of a SSSC-based stabilizer is presented as a QMP problem. The Method presented in [15,16] is expanded and constrains of the problem, in addition to a phase-lead structure, are calculated for a phase-lag structure. By the proposed method, effect of the SSSC-based stabilizer in the magnitude control and the phase control channels on damping inter-area oscillations for a large and a small multi-machine power system has been analyzed and compared.

2. Power system model

2.1. Generator

In this study, the generators are represented by fourth-order d-q axis model. In this case, nonlinear dynamic equations for ith generator are [17]:

\[
\delta_i = \omega_i - \omega_s
\]

\[
M_i \dot{\omega}_i = P_{im} - P_{ni} - D \left( \frac{\omega_i}{\omega_s} - 1 \right)
\]

\[
T_{d} \dot{E}_{di} = -E_{di} - (X_{di} - X_{dq})I_{di} + E_{qi}
\]

\[
T_{q} \dot{E}_{qi} = -E_{qi} + (X_{dq} - X_{di})I_{qi}
\]

\[
P_a = (I_{di}E_{di} + I_{qi}E_{qi}) + (X_{df} - X_{dq})I_{di}I_{qi}
\]

2.2. Exciter

The IEEE type-AC-4A excitation system [18] is considered in this work. Its block diagram is shown in Fig. 1.

2.3. SSSC modeling

It is assumed that, in an n-machine power system, a SSSC is installed on the transmission line between nodes 1 and 2 as shown in Fig. 2.

The SSSC consists of a series coupling transformer (SCT) with the leakage reactance $X_{SC}$, a voltage source converter (VSC) based on gate-turn-off-thyristors (GTOs) and a DC capacitor. The SSSC can be described as [12]:

\[
V_{inj} = mkV_{dc}(\cos \phi + j \sin \phi)
\]

\[
I_L = I_D + I_Q = |I_L|e^{j\psi}
\]

\[
\frac{dV_{dc}}{dt} = \frac{mk}{C_{dc}}(I_D \cos \phi + I_Q \sin \phi)
\]

where $V_{inj}$ is the ac injected voltage by the SSSC; $m$ and $\phi$ are the modulation ratio and phase defined by pulse width modulation (PWM), respectively; $k$ is the ratio between the ac and dc voltage depending on the converter structure; $V_{dc}$ is the dc voltage; $C_{dc}$ is the dc capacitor value, and $I_D$ and $I_Q$ are D- and Q-components of the line current $I_L$, respectively.

3. SSSC-based stabilizers

3.1. Phase-based stabilizer

Assuming a lossless SSSC, the ac voltage is kept in quadrature with the line current so that the SSSC only exchanges reactive power with the transmission line. By adjusting the magnitude of the injected voltage, the reactive power exchange can be controlled. When the SSSC voltage lags the line current by 90°, it emulates a series capacitor. It can also emulate a series inductor when the voltage leads the line current by 90°. Thus, a SSSC can be considered as a series reactive compensator where the degree of compensation can be varied by controlling the magnitude of the injected voltage. In this paper, the SSSC is considered in capacitor mode. To keep the injected voltage in quadrature with the line current, a PI controller, as shown in Fig. 3, has been used. Here $\phi_{inj}$ is the phase of the injected voltage in steady-state and its value is considered as $\phi_{inj} = -90° + \psi_{sc}$.

$\psi_{sc}$ is the angle of the line current in steady-state; $T_{SSSC}$ is time constant of the converter, and $K_P$ and $K_I$ are the proportional and integral gain of PI controller, respectively. A lead-lag stabilizer for damping inter-area oscillations is included in the PI controller. In this case, the stabilizer is called the phase-based stabilizer and for convenience in this paper, it is called $\phi$-based stabilizer. In this stabilizer, $T_w$ is washout time constant; $T_{SSSC}$ is time constant, and $x_2$, $x_1$, and $x_0$ are parameters to be determined. The feedback signal for the stabilizer is selected among local signals as the line-current, the line-real power, and the line-reactive power.

3.2. Magnitude-based stabilizer

To control the magnitude of the injected voltage, modulation ratio $m$ can be controlled. Fig. 4 shows the block diagram of the controller in this case, where $m_{ref}$ is the value of modulation ratio in steady-state. A stabilizer for damping of inter-area oscillations is included in the magnitude controller. This stabilizer is called $m$-based stabilizer.

4. Stabilizer design

The method adopted in this work is an extension of proposed method in [15,16] taking into account phase-lag stabilizer for SSSC.
This method is summarized as follows. In the first step, the closed-loop system is considered as in Fig. 5, where \( G(s) \) and \( F(s) \) are the power system transfer matrix and the stabilizer transfer matrix, respectively. In the second step the stabilizer transfer matrix is changed by \( \Delta F \). In this case, the closed-loop system changes as shown in Fig. 6, where \( G(s) \) is the transfer matrix of the inner loop between \( G(s) \) and \( F(s) \).

In these figures, \( U_{\text{ref}} \) is considered as the input signal of the system. In the phase-based stabilizer \( U_{\text{ref}} \) is replaced by \( V_{\text{dc ref}} \) and in the magnitude-based stabilizer it is replaced by \( m_{\text{ref}} \). One of the local signals is selected as the feedback signal for stabilizer. The value of \( \Delta F \) is calculated to shift eigenvalues of critical modes to desired values.

In the following, a procedure for calculation of \( \Delta F \) is presented. Assuming that variations of \( \Delta F \) is sufficiently small, the variation of the eigenvalue \( \lambda_i \) can be approximated as

\[
\Delta \lambda_i = \rho_i \Delta F(\lambda_i) \quad (i = 1, 2, \ldots, n)
\]  

(9)

where \( \rho_i \) is the residue associated to the ith eigenvalue \( \lambda_i \) of \( G(s) \). Eq. (9) can be rewritten as

\[
\Delta \lambda_i = |\text{Re}(\rho_i)| |\text{Re}(\Delta F(\lambda_i))| - |\text{Im}(\rho_i)||\text{Im}(\Delta F(\lambda_i))| + j|\text{Re}(\rho_i)||\text{Im}(\Delta F(\lambda_i))| - |\text{Im}(\rho_i)||\text{Re}(\Delta F(\lambda_i))|
\]

(10)

It is assumed that the stabilizer has a lead-lag structure as follows:

\[
f(s) = \frac{x_0 s^2 + x_1 s + x_0}{(1 + s)^2} \frac{s T_w}{1 + s T_w}
\]

(11)

By substituting \( s = \lambda_i \) and \( x_0 = \Delta \lambda_i \) in (11), the real and imaginary part variations of \( \Delta F(\lambda_i) \) are

\[
\text{Re}(\Delta F(\lambda_i)) = R_{\theta 1} \Delta \lambda_2 + R_{\theta 2} \Delta \lambda_1 + R_{\theta 0} \Delta \lambda_0
\]

(12)

\[
\text{Im}(\Delta F(\lambda_i)) = l_{\theta 1} \Delta \lambda_2 + l_{\theta 2} \Delta \lambda_1 + l_{\theta 0} \Delta \lambda_0
\]

(13)

where \( R_{\theta 0}, R_{\theta 1}, R_{\theta 2}, l_{\theta 0}, l_{\theta 1}, \) and \( l_{\theta 2} \) are specified values. To shift critical eigenvalues to the left of the imaginary axis, we must have

\[
\text{Re}(\Delta \lambda_i) \leq -|\Delta \sigma_i|
\]

(14)

\[
-|\Delta \omega_0| \leq |\text{Im}(\Delta \lambda_i)| \leq |\Delta \omega_0|
\]

(15)

where \( \Delta \sigma_i \) and \( \Delta \omega_0 \) are the desired shift value of the real part and acceptable frequency variations of the critical eigenvalue \( \lambda_i \), respectively. Substituting (12) and (13) in (10), we can obtain \( \Delta \lambda_i \) as the linear function from \( \Delta \lambda_0, \Delta \lambda_1 \) and \( \Delta \lambda_2 \). Then, substituting the real part of \( \Delta \lambda_i \) in (14) and its imaginary part in (15) yields

\[
x_0 \Delta \lambda_2 + x_1 \Delta \lambda_1 + x_0 \Delta \lambda_0 \leq -|\Delta \sigma_i|
\]

(16)

\[
-|\Delta \omega_0| \leq |\Delta \omega_2| + |\Delta \omega_1| + |\Delta \omega_0| \leq |\Delta \omega_0|
\]

(17)

where \( x_0, x_1, x_2, \) and \( l_{\theta 0}, l_{\theta 1}, l_{\theta 2} \) are specified values. On the other hand, if the angle of residue \( \rho_i \) is positive, the stabilizer must have a phase-lead characteristic; otherwise, it must have a phase-lag characteristic. Considering \( x_2 = x_2/(\omega_0 T_s^2), x_1 = x_1/(\omega_0 T) \) and \( s = sT \) and substituting in (11) yields

\[
f(s) = x_0 \frac{x_2 s^2 + x_1 s + x_0}{(1 + s)^2} \frac{T_w s}{T + T_w s}
\]

(18)
Function \( \tilde{f}(s) \) has a double pole at \( s = -1 \). According to [19] for a phase-lead structure, it is assumed that zeroes of \( \tilde{f}(s) \) are almost one-decade nearer to the center than that of its poles, i.e., they belong to the interval \([-1 -0.1]\) and for the phase-lag structure the zeroes are almost one-decade farther to the center, i.e., they belong to interval \([-10 -1]\). This subject is graphically shown for lead and lag structure in Figs. 7 and 8, respectively. In this figures the points inside the triangles and above the parabolas correspond to above intervals. To represent the constraints as a linear function, the parabolas are approximated by lines. In this case the zeros may be as complex values, but real parts of the complex zeros are located in the above intervals.

As illustrated in Fig. 8, the constraints for phase-lag stabilizer are

\[
\begin{align*}
\dot{x}_1 - \dot{x}_2 &\leq 1 \\
10x_1 - 100\dot{x}_2 &\leq 1 \\
-99\dot{x}_1 + 180\dot{x}_2 &\leq -18
\end{align*}
\]

Substituting \( \dot{x}_2 \) and \( \dot{x}_1 \) in (19)-(24) and assuming \( x_0 = \Delta x_1 + x_2 \) (\( z = 1, 2, 3 \)) where \( \dot{x}_0, \dot{x}_1 \) and \( \dot{x}_2 \) are known values, the constraints can be rewritten for lead structure as in (25)-(27) and for lag structure as in (28)-(30).

\[
\begin{align*}
-\Delta x_2 + T\Delta x_1 - T^2\Delta x_0 &\leq \dot{x}_2 - T\dot{x}_1 + T^2\dot{x}_0 \\
-\Delta x_3 + 10T\Delta x_1 - 100T^2\Delta x_0 &\leq \dot{x}_2 - T\dot{x}_1 + T^2\dot{x}_0 \\
18\Delta x_2 - 99T\Delta x_1 + 180T^2\Delta x_0 &\leq -18\dot{x}_2 + 99T\dot{x}_1 - 180T^2\dot{x}_0 \\
-\Delta x_2 + T\Delta x_1 - T^2\Delta x_0 &\leq \dot{x}_2 - T\dot{x}_1 + T^2\dot{x}_0 \\
100\Delta x_2 + 10T\Delta x_1 - T^2\Delta x_0 &\leq +100\dot{x}_2 - T\dot{x}_1 + T^2\dot{x}_0 \\
180\Delta x_2 - 99T\Delta x_1 + 180T^2\Delta x_0 &\leq -180\dot{x}_2 + 99T\dot{x}_1 - 180T^2\dot{x}_0
\end{align*}
\]

The following function, as the gain of the stabilizer at the frequency \( \omega_p \), \( p = 1, 2, \ldots, N \), is considered as the objection function [16]. As a criterion, to achieve a same damping ratio for a critical oscillation mode, the stabilizer with lower loop gain is more valuable.

\[
J = \min \sum_{p=1}^{N} |f(j\omega_p)|^2
\]

where \( \omega_1, \omega_2, \ldots, \omega_N \) are a set of frequencies in the region where the critical eigenvalue must be shifted. By substituting (11) in (31) and considering \( s = j\omega \) and \( X = \dot{X} + \Delta X \) we can easily rewrite the objective function as

\[
J = \min \frac{1}{2} \Delta X^T H \Delta X + f^T \Delta X
\]

where the matrix \( H \), vector \( f \) and vector \( \dot{X} \) are known and \( \Delta X \) is the unknown vector to be tuned.

\[
\dot{X} = [\dot{x}_2 \quad \dot{x}_1 \quad \dot{x}_0]^T
\]

\[
\Delta X = [\Delta x_2 \quad \Delta x_1 \quad \Delta x_0]^T
\]

The objective function (32) with constraints (16), (17), (25)-(30) is as a quadratic mathematical programming problem. The proposed approach is an iterative method. In this method calculated vector of \( \Delta X \) is added to the known vector \( X \) and it is considered as known vector in the next iteration. The vector \( \dot{X} \) in the first iteration is set to be zero. To solve this problem the quadprog algorithm provided by the Matlab Optimization Toolbox is applied here [20]. Matlab is a high-performance language for technical computing. Because of having suitable mathematical library and toolboxes, it is widely used in power systems. Dynamic stability of a multimachine power system, even high order, can easily be studied by the software.

5. Simulation results

The cases studied here are 2-area 4-machine and 2-area 50-machine power systems as small and large power systems, respectively.

5.1. The IEEE 4-machine power system

A single line diagram of the system is shown in Fig. 9. Data of this system is represented in [21]. To control inter-area oscillations, a SSSC is installed in the tie-line between nodes 5 and 6. The loads are modeled as constant impedances. Specific parameters used for the SSSC are considered as \( T_{SSSC} = 0.01 \) s, \( k = 1 \) pu, \( X_{SAC} = 0.15 \) pu, \( C_{dl} = 1 \) pu, \( V_{dref} = 1 \) pu, \( K_p = 25 \) pu and \( K_i = 200 \) pu.
In this system, three operating scenarios based on the transmitted power between the interconnected areas are investigated. To increase transferred power, the load in area 2 has been increased and the load in area 1 has then been modified to achieve a given tie-line transferred power. The summary of operating conditions that are studied in this paper is shown in Table 1.

Open-loop oscillation modes in different operating conditions are shown in Table 2. The inter-area mode is shown with bold values in this table. It is clear that increasing transmitted power damping ratio of inter-area mode is noticeably reduced and close to instability point.

A SSSC stabilizer to improve the damping of inter-area oscillations is designed. For comparison, the parameters of the $\phi$-based and the $m$-based stabilizers are calculated. According to the designed method, a signal with the maximum residue for the inter-area mode is selected as the feedback signal for the damping stabilizer. Table 3 shows residues for the inter-area mode for different operating conditions. It can be seen from this table that:

(i) When the stabilizer is applied for the phase control channel (the $\phi$-based stabilizer), the residues for the inter-area mode for different local feedback signals are higher than that of the case when the stabilizer is used for the magnitude control channel (the $m$-based stabilizer).

(ii) Variation of the current in the transmission line, where the SSSC is installed, is the best signal for both stabilizers.

It is assumed that the desired damping ratio for the inter-area mode in each operating condition is $\zeta = 0.25$. According to the frequency of oscillation modes in the open-loop system, the value of $\omega_p$ in (31) has been considered $\omega_p = 1.3, 3.5$. The time constant of stabilizer is set as $T = 0.4$. Also, the effects of stabilizer on other modes must be considered so that the damping ratio of other modes has been increased or does not become less than a specific value and the variations of their frequencies be acceptable. To improve the damping ratio of inter-are mode to a desired value, the parameters of the proposed stabilizers are calculated and shown in Table 4. This table shows that for the same desired damping ratio, the parameters of the $\phi$-based stabilizer are less than that of the $m$-based stabilizer. In this table the loop gains of stabilizers in the frequency of the inter-area mode $|\{\omega_p\}|$ are shown. These values show that to achieve the same damping ratio, the gains of the $\phi$-based stabilizer are less than the gains of the $m$-based stabilizer. In other words, the $\phi$-based stabilizer is more effective for the damping of inter-area oscillations.

Oscillation modes of the system in the closed-loop for different operating conditions for both stabilizers are shown in Table 5. Comparing Tables 2 and 5 shows that the damping ratio of the inter-area mode has been improved to the desired value. In addition, it can be seen that other modes have not been degraded significantly.

For completeness and verification of the designed stabilizers, a three-phase fault was applied to the test system at bus 6. The fault is cleared without line switching. Because generators 1 and 3 have the most contribution in the inter-area mode, typically, the swing angle of generator 1 with respect to generator 3, in heavy loading, for the $\phi$-based stabilizer and the $m$-based stabilizer is shown in Fig. 10. This figure shows that SSSC stabilizers can effectively damp inter-area oscillations. Control signals of damping stabilizers in heavy loading are shown in Fig. 11. It can be seen that the $\phi$-based stabilizer provides much less control effort compared to that of the $m$-based stabilizer.

5.2 IEEE 50-machine power system

The 50-machine power system is a moderate sized system that includes all the modeling features and the complexity of a

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**Table 1**
The summary of operating conditions in the 4-machine power system.

<table>
<thead>
<tr>
<th>Operating conditions</th>
<th>Transmitted power (MW)</th>
<th>Load of area 1 (MW)</th>
<th>Load of area 2 (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>180</td>
<td>1120</td>
<td>1180</td>
</tr>
<tr>
<td>Normal</td>
<td>380</td>
<td>920</td>
<td>1380</td>
</tr>
<tr>
<td>Heavy</td>
<td>410</td>
<td>890</td>
<td>1410</td>
</tr>
</tbody>
</table>

**Table 2**
Open-loop oscillation modes in the 4-machine power system.

<table>
<thead>
<tr>
<th>Light loading modes</th>
<th>Normal loading</th>
<th>Heavy loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1.112 \pm j7.741$</td>
<td>$-1.142 \pm j7.756$</td>
<td>$-1.243 \pm j7.744$</td>
</tr>
<tr>
<td>$-1.812 \pm j7.456$</td>
<td>$-1.809 \pm j7.541$</td>
<td>$-1.539 \pm j7.534$</td>
</tr>
<tr>
<td>$-1.379 \pm j1.040$</td>
<td>$-0.532 \pm j2.451$</td>
<td>$-0.436 \pm j2.722$</td>
</tr>
<tr>
<td>$-0.054 \pm j1.068$</td>
<td>$-1.161 \pm j1.873$</td>
<td>$-1.294 \pm j0.857$</td>
</tr>
<tr>
<td>$0.436 \pm j2.924$</td>
<td>$-0.183 \pm j1.640$</td>
<td>$-0.0469 \pm j1.838$</td>
</tr>
<tr>
<td>$-0.903 \pm j1.068$</td>
<td>$-1.119 \pm j0.732$</td>
<td>$-0.584 \pm j0.827$</td>
</tr>
</tbody>
</table>

**Table 3**
Residues for inter-area mode for different signals in the 4-machine power system.

<table>
<thead>
<tr>
<th>Operating conditions</th>
<th>SSSC-based stabilizer</th>
<th>Input signal</th>
<th>$A_f$</th>
<th>$A_Q$</th>
<th>$A_P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>$\phi$-based</td>
<td>1.06</td>
<td>0.97</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\leq 75.230$</td>
<td>$\leq 75.480$</td>
<td>$\leq 74.320$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$m$-based</td>
<td>0.65</td>
<td>0.61</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\leq 84.920$</td>
<td>$\leq 84.440$</td>
<td>$\leq 83.610$</td>
<td></td>
</tr>
<tr>
<td>Normal</td>
<td>$\phi$-based</td>
<td>3.57</td>
<td>2.04</td>
<td>3.18</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\leq 110.20$</td>
<td>$\leq 126.69$</td>
<td>$\leq 105.49$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$m$-based</td>
<td>0.75</td>
<td>0.43</td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\leq 112.40$</td>
<td>$\leq 95.910$</td>
<td>$\leq 117.11$</td>
<td></td>
</tr>
<tr>
<td>Heavy</td>
<td>$\phi$-based</td>
<td>3.46</td>
<td>1.16</td>
<td>3.23</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\leq 153.23$</td>
<td>$\leq 176.38$</td>
<td>$\leq 151.76$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$m$-based</td>
<td>0.64</td>
<td>0.28</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\leq 105.03$</td>
<td>$\leq 87.430$</td>
<td>$\leq 106.16$</td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 9.** One-line diagram of the 4-machine power system.
large-scale power system. This system contains 44 generators represented by a classical model and six generators represented by a two-axis model and equipped with an exciter. No PSS is installed in the system. The system data is given in [22]. Loads are modeled as constant impedances. In this paper, the operating conditions are characterized by increasing the real power generation at buses 93 and 110 without any variation in loads of the system. For several operating conditions, the load flow is done and eigenvalues of the system are calculated. Table 6 shows inter-area modes of this system for each operating conditions. By increasing the generation level at buses 93 and 110, it can be seen that the damping ratio of inter-area mode 1 with the frequency around 0.42 Hz decreases but the damping ratio of inter-area mode 2 with a frequency close to 0.65 Hz is approximatively constant. Therefore, inter-area mode 1 is considered as a critical mode and the goal of the SSSC is to improve damping of the mode.

In this system, the variation of the line current magnitude is considered as the feedback signal. The best installing location for SSSC is selected by the residue analysis [23]. By calculating residues in different locations in the studying system, the line 66–63 is selected as optimal location for the SSSC. The used data for SSSC in the present study are considered the same as used for SSSC in the 4-machine power system.

The purpose of SSSC-based stabilizer in this power system is to increase the damping ratio of inter-area mode 1 to a maximum possible value. In (31), we set $\omega = 1.3, 5$ and $T = 0.4$. Typically, parameters and the loop gains of stabilizers at the frequency of inter-area mode 1 for the two generation levels 1000 and 1600 MW at Buses 93 and 110 are shown in Table 7. The closed-loop inter-area modes are shown in Table 8. It is clear from these tables that the $\phi$-based stabilizer, in comparison to the $m$-based stabilizer, can

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### Table 4

Parameters of the proposed stabilizes in the 4-machine power system.

<table>
<thead>
<tr>
<th>Operating conditions</th>
<th>$m$-Based stabilizer</th>
<th>$\phi$-Based stabilizer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x_2$</td>
<td>$x_1$</td>
</tr>
<tr>
<td>Light</td>
<td>0.1402</td>
<td>0.3856</td>
</tr>
<tr>
<td>Normal</td>
<td>0.1865</td>
<td>0.2626</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.3923</td>
<td>0.3713</td>
</tr>
</tbody>
</table>

### Table 5

Closed-loop oscillation modes in the 4-machine power system.

<table>
<thead>
<tr>
<th>Light loading</th>
<th>$m$-Based stabilizer</th>
<th>$\phi$-Based stabilizer</th>
<th>Normal loading</th>
<th>$m$-Based stabilizer</th>
<th>$\phi$-Based stabilizer</th>
<th>Heavy loading</th>
<th>$m$-Based stabilizer</th>
<th>$\phi$-Based stabilizer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-1.127 \pm j7.744$</td>
<td>$-1.127 \pm j7.744$</td>
<td></td>
<td>$-1.159 \pm j7.757$</td>
<td>$-1.142 \pm j7.756$</td>
<td></td>
<td>$-1.273 \pm j7.719$</td>
<td>$-1.242 \pm j7.745$</td>
</tr>
<tr>
<td></td>
<td>$-1.828 \pm j7.457$</td>
<td>$-1.827 \pm j7.455$</td>
<td></td>
<td>$-1.823 \pm j7.529$</td>
<td>$-1.808 \pm j7.541$</td>
<td></td>
<td>$-1.561 \pm j7.506$</td>
<td>$-1.559 \pm j7.534$</td>
</tr>
<tr>
<td></td>
<td>$-1.398 \pm j0.935$</td>
<td>$-1.352 \pm j1.000$</td>
<td></td>
<td>$-1.516 \pm j2.315$</td>
<td>$-1.218 \pm j2.259$</td>
<td></td>
<td>$-0.478 \pm j2.658$</td>
<td>$-0.297 \pm j2.600$</td>
</tr>
<tr>
<td></td>
<td>$-0.054 \pm j1.065$</td>
<td>$-0.054 \pm j1.065$</td>
<td></td>
<td>$-0.463 \pm j2.346$</td>
<td>$-0.291 \pm j2.361$</td>
<td></td>
<td>$-1.196 \pm j0.659$</td>
<td>$-1.333 \pm j1.172$</td>
</tr>
<tr>
<td></td>
<td>$-0.862 \pm j3.336$</td>
<td>$-0.829 \pm j3.202$</td>
<td></td>
<td>$-0.373 \pm j1.443$</td>
<td>$-0.372 \pm j1.405$</td>
<td></td>
<td>$-0.396 \pm j1.532$</td>
<td>$-0.481 \pm j1.840$</td>
</tr>
<tr>
<td></td>
<td>$-0.842 \pm j1.096$</td>
<td>$-0.883 \pm j1.087$</td>
<td></td>
<td>$-1.097 \pm j0.706$</td>
<td>$-1.371 \pm j0.786$</td>
<td></td>
<td>$-0.581 \pm j0.827$</td>
<td>$-0.562 \pm j0.915$</td>
</tr>
</tbody>
</table>

### Table 6

Open-loop inter-area modes and their damping ratio in the 50-machine power system.

<table>
<thead>
<tr>
<th>Generation level at G93, G110 (MW)</th>
<th>Inter-area mode 1</th>
<th>Inter-area mode 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>$-0.043 \pm j2.809$ ($0.0153$)</td>
<td>$-0.203 \pm j4.107$ ($0.0494$)</td>
</tr>
<tr>
<td>1200</td>
<td>$-0.031 \pm j2.753$ ($0.0112$)</td>
<td>$-0.201 \pm j4.097$ ($0.0490$)</td>
</tr>
<tr>
<td>1400</td>
<td>$-0.017 \pm j2.686$ ($0.0063$)</td>
<td>$-0.198 \pm j4.084$ ($0.0484$)</td>
</tr>
<tr>
<td>1600</td>
<td>$0.002 \pm j2.603$ ($-0.0007$)</td>
<td>$-0.195 \pm j4.068$ ($0.0478$)</td>
</tr>
</tbody>
</table>
effectively improve the damping of the critical inter-area mode. Also, it can be seen that the $\phi$-based, having more effects on the critical inter-area mode, has a loop gain close to the $m$-based stabilizer.

To demonstrate the effectiveness of the proposed stabilizers in the 50-machine power system, a three-phase fault was applied at bus 7 for duration of 0.03 s. The fault is cleared without the line tripping. Because generators 93, 95 and 145, have the most contribution in the oscillations of the critical mode, typically, the swing angle of generator 93 with respect to generator 145 when the generation level at buses 93 and 110 is set to 1600 MW, is shown in the Fig. 12. It can be seen that the SSSC damping stabilizer, particularly in the angle control channel, can effectively damp inter-area oscillations.

A comparative study between this paper and previous study in [15,16] to design PSS show that, for power systems in the presence of a SSSC stabilizer to improve damping inter-area oscillations, the stabilizer may need to a lag structure instead of leading structure. Also, results of this paper show that to achieve desirable damping ratio for inter-area oscillations, the size and consequently the control cost of the SSSC stabilizer in the phase control channel are lower in comparison to the magnitude control channel presented in [11]. The influence of the stabilizer parameters for different control channels on the transient stability limit will be undertaken for further investigation.

6. Conclusion

In this paper damping of inter-area oscillations by SSSC has been studied. Suitable location for installing SSSC and the best feedback signal for damping stabilizer are selected by residues analysis. A method by quadratic mathematic programming is presented to design damping stabilizer. Using this method influence of SSSC stabilizers for different control channels on damping of inter-area oscillations is investigated. Obtained results on a small and a large multi-machine power system under different operating conditions show that when the SSSC stabilizer is used for phase control channel, comparing the magnitude control channel, is more effective for damping of inter-area oscillations.

References


