Direct solution to problems of static sharp waves in shallow-water

R. Mansouri\(^a\)*, F. Oimdi Nasib\(^b\), A.H. Haghiabi\(^c\) B. Morshed Zadeh\(^d\)

\(^a\)Ph. D. Student; Water Eng. Dep., Lorestan University, KhoramAbad, Iran.
\(^b\)Assist. Prof., Water Eng. Dep., Lorestan University, KhoramAbad, Iran.
\(^c\)Assoc. Prof., Water Eng. Dep., Lorestan University, KhoramAbad, Iran.
\(^d\)Ph.D. Student; Management, Tehran University, Tehran, Iran.

*Corresponding author; Ph. D. Student; Water Eng. Dep., Lorestan University, KhoramAbad, Iran.

ARTICLE INFO

Article history
Received 01 April 2014
Accepted 19 April 2014
Available online 28 April 2014

Keywords,
The static sharp wave
Shallow-water
Taylor series
Euler's equation
Differential equations
Potential speed
Wave height

ABSTRACT

The study of hydrodynamic canals is the first step in canals design, sediment transport, erosion, dissemination of pollution and other phenomena related to canals. When in canals, depth or flow rate is suddenly changed, the sharp wave is generated. When the position and characteristics of the sharp wave remain constant, after the steady flow, it is called the static-sharp wave. So generated jump hydraulic and sharp waves in transitions due to cross-obstacle in flow path at supercritical flows are classified as the static sharp waves. Since the equations governing the dynamics of sharp waves are the same as the flow equation in shallow-water, this paper uses linear solve for the equations governing shallow-water and specifying boundary conditions in shallow-water with the static wave and by using Taylor series, an equation for the bottom and two equations based on kinematic and dynamic boundary for the surface boundary were extracted. Also considering the frequency of the surface waves, frequency boundary conditions considering introduction of the dimensionless parameter \(\theta\) based on wave number and angular frequency, were extracted and other boundary conditions were rewritten based on the dimensionless parameter \(\theta\). Next, based on the velocity potential and the Laplace equation, generated the differential equations solved by Euler equation, which leads to generate potential velocity \((\phi)\) and wave height \((\eta)\) is the canal length.
1. Introduction

The study of hydrodynamic canals is the first step in canals design, sediment transport, erosion, dissemination of pollution and other phenomena related to canals.

An important part of the hydrodynamic is focusing on to the flow in canals and investigating the causes of generating and the pattern of this. One of the most important hydrodynamic parameters of the canal is waves generated.

One of the most comprehensive simulation and prediction of waves can be achieved by using analytical methods, because the hydraulic simulation models, have computational limitations. So it limits possibility of applying in small areas.

When in canals, depth or flow rate is suddenly changed, the sharp wave is generated.

Mainly, sharp waves are generated in super critical flow, however flow from sub-critical to super critical is a sharp wave that is called hydraulic jump. If the location and characteristics of sharp wave changes with time, or sharp wave moves, it is called dynamic-sharp waves. For example, the generated waves in dam failure or sudden opening and closing of valves are dynamic-sharp waves. On the other side, if the position and characteristics of the sharp wave, after the steady flow, remain constant, it is called the static-sharp wave. So generated jump hydraulic and sharp wave in transitions due to cross-obstacle in flow path at supercritical flows are classified as the static sharp waves. The equations governing the dynamics of sharp waves are the same as the flow equation in shallow-water.

Theoretical aspects of the problem of small-amplitude water waves travelling in a region of varying depth, mainly uniqueness results, have been presented, under various geometric assumptions, by Vainberg & Maz'ja (1973), Fitz-Gerald (1976), Fitz-Gerald & Grimshaw (1979), Simon & Ursell (1984), Kuznetsov (1991, 1993) and other authors. See also the survey by Evans & Kuznetsov (1997). Besides, general methods for direct numerical solution of the linear problem, such as finite-element methods, boundary-integral-equation methods or hybrid techniques are available; see, e.g. the surveys by Mei (1978, 1983), Euvrard et al. (1981), Yeung (1982), Porter & Chamberlain (1997). Moreover, general numerical techniques based on a topography discretization or on a domain transformation have been developed. See, e.g. Devillard, Dunlop & Souillard (1988), Rey (1992) for the former, and Evans & Linton (1994) for the latter approach. However, the computational cost of these generic techniques is high, rendering them inappropriate, especially for long-range propagation and three dimensions. Owing to this fact, a constantly growing emphasis has been given on the development of approximate wave models retaining only the essential features of specific families of problems and, thus, being better suited for long-range wave propagation.

A well-known specific feature of water waves is that the propagation space does not coincide with the physical space. While the latter is the whole liquid domain (an irregularly shaped horizontal strip in the case of a shallow-sea environment), the former is only the horizontal direction(s). This fact, which is a manifestation of the surface (boundary) character of water waves, leads to the reformulation of the propagation problem as a non-local wave equation in the propagation (horizontal) space. In the linear case, on which we shall focus our attention in the present work, the appropriate non-local equation may take the form (in the time domain) of either an operator differential equation (Garipov 1965; Moiseev 1964; Moiseev&Rumiantsev1968; Craig & Sulem 1993), or an Integro differential equation (Fitz-Gerald 1976), or a pseudo differential equation (Milder 1977; Miles 1977; Craig &Sulem 1993) or even a partial differential equation with respect to the complex-analytic wave potential(Athanassoulis & Makrakis 1994). Another possibility, which will be discussed in detail in the present work, is to reformulate the problem as an infinite system of horizontal equations with variable coefficients. A clear consequence of the non-local character of the water-wave problem in the propagation space is that any one-equation model (i.e. one deferential equation in the horizontal direction(s)) cannot capture all features of the problem. Nevertheless, because of the complexity of the problem, a plethora of one-equation models have been proposed and studied, each one with its range of applicability and its advantages.