The spectral characterizations of the connected multicone graphs $K_w \triangleleft LHS$ and $K_w \triangleleft LGQ(3, 9)$

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In the past decades, graphs that are determined by their spectrum have received much more and more attention, since they have been applied to several fields, such as randomized algorithms, combinatorial optimization problems and machine learning. An important part of spectral graph theory is devoted to determining whether given graphs or classes of graphs are determined by their spectra or not. So, finding and introducing any class of graphs which are determined by their spectra can be an interesting and important problem. The main aim of this study is to characterize two classes of multicone graphs which are determined by their adjacency, Laplacian and signless Laplacian spectra. A multicone graph is defined to be the join of a clique and a regular graph. Let $K_w$ denote a complete graph on $w$ vertices. In the paper, we show that multicone graphs $K_w \triangleleft LHS$ and $K_w \triangleleft LGQ(3, 9)$ are determined by both their adjacency spectra and their Laplacian spectra, where $LHS$ and $LGQ(3, 9)$ denote the Local Higman-Sims graph and the Local GQ(3, 9) graph, respectively. In addition, we prove that these multicone graphs are determined by their signless Laplacian spectra.

Keywords: DS graph; multicone graph; adjacency spectrum; Laplacian spectrum; signless Laplacian spectra; Local Higman–Sims graph; Local GQ(3, 9) graph.

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1. Introduction

We begin with some of the notations and terminology that will be used in the paper. All graphs considered here are simple and undirected. All terminology and notations on graphs which are not defined here can be found in [2, 4, 14, 32]. Let $\Gamma = (V, E)$ be a simple graph, where $V$ is the set of vertices and $E$ is the set of edges of $\Gamma$. An edge joining the vertices $u$ and $v$ is denoted by $\{u, v\}$. The complement or inverse of a graph $G$ is a graph $H$ on the same vertices such that two distinct vertices of $H$ are adjacent if and only if they are not adjacent in $G$. The complement of a graph $G$ is denoted by $\overline{G}$. The union of two disjoint graphs $G_1$ and $G_2$ is denoted by $G_1 \cup G_2$, is the graph whose vertices (respectively, edges) set is the union of vertices (respectively, edges) sets of $G_1$ and $G_2$. The join of two disjoint graphs $G_1$ and $G_2$ is the graph that is obtained from $G_1 \cup G_2$ by joining each vertex in $G_1$ with every vertex in $G_2$. It is denoted by $G_1 \vee G_2$.

A graph is called bigrideed if the set of degrees of vertices consists of exactly two distinct elements. Let $A(G)$ be the $(0, 1)$-adjacency matrix of graph $G$. The characteristic polynomial of $G$ is $\det(\lambda I - A(G))$, and is denoted by $P_G(\lambda)$. The roots of $P_G(\lambda)$ are called the adjacency eigenvalues of $G$ and since $A(G)$ is real and symmetric, the eigenvalues are real numbers. If $G$ has $n$ vertices, then it has $n$ eigenvalues in descending order as $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$. Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the distinct eigenvalues of $G$ with multiplicities $m_1, m_2, \ldots, m_n$, respectively. The multi-set $\text{Spec}_A(G) = \{[\lambda_1]^{m_1}, [\lambda_2]^{m_2}, \ldots, [\lambda_n]^{m_n}\}$ of eigenvalues of $A(G)$ is called the adjacency spectrum of $G$. For two graphs $G$ and $H$, if $\text{Spec}_A(G) = \text{Spec}_A(H)$, we say $G$ and $H$ are cospectral with respect to adjacency matrix. A graph $H$ is said to be determined by its spectrum or DS for short, if for any graph $H$ with $\text{Spec}_A(G) = \text{Spec}_A(H)$, one has $G$ isomorphic to $H$. The maximum eigenvalue of a graph $G$ is called the spectral radius and it is denoted by $\rho(G)$. Let $G$ be a graph with adjacency matrix $A$ and $D$ be the diagonal matrix of vertex degrees for $G$. The matrices $Q = SL(G) = D + A$ and $L(G) = D - A$ are known as the signless Laplacian matrix and the Laplacian matrix for $G$, respectively. In mathematical graph theory, the Higman–Sims graph is a 22-regular undirected graph with 100 vertices and 1100 edges. It is the unique strongly regular graph with 100 vertices and valency 22, where no neighboring pair of vertices share a common neighbor and each non-neighboring pair of vertices share six common neighbors. The Local Higman–Sims graph is a 20-regular undirected graph with 81 vertices and 810 edges. The Local $GQ(3, 9)$ graph is a 20-regular undirected graph with 81 vertices and 810 edges.

So far numerous examples of cospectral but non-isomorphic graphs have been constructed by interesting techniques such as Seidel switching, Godsil–McKay switching, Sunada or Schwenk method. For more information, one may see [9, 27, 28] and the references cited in them. Only a few graphs with very special structures have been reported to be determined by their spectra (DS, for short) (see [8, 11, 12, 14, 15, 17, 20, 21, 29, 30] and the references cited in them). Recently, Wang and Xu have developed a new method in [30] to show that many graphs are...