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Proliferated integration of variable renewable resources, especially wind power, in multi-area power systems brings about critical needs to coordinate balancing resources over the whole interconnected system. This paper proposes a coordination methodology in the real-time electricity market between regional interconnected power systems. The new method optimally dispatches generating units, determines interregional energy trading, and allocates regulation reserve throughout the entire system. The balancing resources in participating areas are shared and more effectively used to tackle the wind power uncertainty by means of a combined automatic generation control system. In order to comply with the existing operating philosophy of regional electricity markets, the augmented Lagrangian relaxation along with alternative direction method of multipliers are applied to provide a fully decentralized procedure with limited data interchanges between neighboring regions. Performance of the proposed method is examined on a two-area 6-bus and the modified three-area IEEE 118-bus test systems. The results confirm the applicability and effectiveness of the proposed model in providing flexible energy trading, regulation sharing, improved economic efficiency, and increased ability to integrate variable renewable energy into the power systems. Published by AIP Publishing. [http://dx.doi.org/10.1063/1.4978305]

I. INTRODUCTION

Large-scale power systems usually consist of several interconnected regional power grids each of which might be a self-governing system with its own operation and control regulations. In each region, power system operators are responsible for maintaining the load-generation balance within their territories as well as for following interchange schedules between regions. The high penetration of renewable variable generations, especially wind power, due to their intermittent nature and imperfect predictability, increases the challenge of preserving the power balance. The challenge becomes more severe when interconnected regions are being operated locally and independently of each other.

Due to the spatial smoothing effect of geographically dispersed wind farms, the impact of variability and uncertainty in wind power generation becomes relatively smaller if these resources are lumped together over the whole interconnected system. According to existing practices, the areas have to individually deal with their own local variable generation during real-time operations. In this way, the potential of wide-area integration of wind power generation and the ability of different regions to cooperate and coordinate balancing resources over the whole interconnected system are limited. Consequently, for an area with limited balancing resources under high penetration of variable generations, the power system operator frequently may have to deploy more expensive resources or sometimes may even run out of resources to maintain balance. However, coordination between the areas in real-time operation can promote more
energy exchange, improve overall economic efficiency, and provide increased ability to integrate variable generations into the power systems. Several research works in both industry and academia have addressed the problem of developing real-time area coordination methods. A few interregional trading projects have been carried out among the U.S. electricity markets to achieve more efficient interchanges and better convergence of prices between regions. These projects include Interchange Optimization by Pennsylvania-New Jersey-Maryland Interconnection (PJM) and Midcontinent Independent System Operator (MISO), Coordinated Transaction Scheduling between NYISO and PJM, and Inter-Regional Interchange Scheduling between New York Independent System Operator (NYISO) and ISO New England (ISONE). Having applied these projects, the interregional trading among neighbouring markets is determined for every 15-min operating intervals. The resulting transactions are then considered as the input data in regional real-time markets. The implementation of this approach has demonstrated the increased convergence of interface prices over the time, less time intervals with inefficient transactions, and reduced uneconomical interregional trading. However, the interregional trading between the independent system operators (ISOs) is not centrally specified and might be sub-optimal. Additionally, the time interval between the interchange optimization and real-time operation may lead to volatile interchanges which should be tackled by an expensive regulation reserve. Since the levels of interchange among the ISOs are not centrally optimized, overall economic dispatch of the several ISOs is not the same as the results of solving a hypothetical ideal integrated model for all ISOs together. However, such a centralized case requires access to data of the whole system, which is undesirable due to difficulties in data management as well as political and technical concerns on information privacy. Accordingly, several decoupling algorithms and distributed optimization methodologies have so far been presented to solve the multi-area economic dispatch, such as decentralized Lagrangian relaxation (LR), augmented Lagrangian relaxation (ALR), auxiliary problem principle, and the alternating direction multiplier method (ADMM) both relying on the augmented Lagrangian, dynamic multiplier-based LR, optimal condition decomposition based on natural decomposition of Karush-Kuhn-Tucker condition, modified generalized Benders’ decomposition technique, and marginal equivalent decomposition method.

In addition to the impact of wind uncertainty on tie-line scheduling and power dispatch in multi-area systems, more net load variations caused by the high penetration of wind power result in more frequency deviation and balancing area interchange violation. To cope with these issues, the areas can share their regulation resources to provide system balance during real-time operation through the automatic generation control (AGC) system. The area control error diversity interchange (ADI) methodology has been proposed to reduce the regulation requirement by combining area control errors (ACEs) from participating areas and sharing the total ACE among all participating areas. The ADI project has been developed by British Columbia Transmission Corporation along with four control areas operated by Idaho Power Company, NorthWestern Energy, and Eastern and Western PacifiCorp control areas. Although the ADI implementation benefits all participating areas, this method may lead to an over-conservative or over-aggressive response and flexibility shortage on control selections. In addition, there is no guarantee that this method achieves the optimal solution found if the whole interconnection is operated by one single balancing area. The authors in Ref. proposed a methodology that uses individual ACEs of each area to approximate the entire system ACE and its distribution over the areas. Moreover, a multi-agent smart generation control scheme for the AGC coordination in interconnected power systems was proposed in Ref. 22.

Despite these recent advances, to the best of our knowledge, coordinated multi-area energy and regulation joint dispatch has not been conducted in the existing literature. To address this issue, this paper proposes a co-optimization model for energy and regulation joint dispatch that effectively coordinates real-time electricity markets of wind-integrated interconnected power systems in a decentralized manner. The main contributions of the proposed model are as follows:

1. A co-optimization model is proposed for simultaneously dispatching generating units, determining interregional energy trading, and allocating regulation reserve in wind-integrated interconnected power systems.
A new coordination methodology between balancing areas is developed to more effectively share balancing resources over the whole interconnected system to tackle variation and uncertainty of system-wide wind power. The proposed model is intended to achieve the optimal solution found if all areas are considered to be one entire balancing area. To do so, the ACE of the entire area is presented by the summation of individual ACEs, and new adaptive sharing factors are introduced to optimally modify the ACEs which are fed into the AGC system of each area.

An area decoupling technique along with a decentralized procedure based on ALR and ADMM are proposed to decompose and parallelize the solution of each regional market. Using this strategy, the market/system operator of each area clears its real-time market independently of the others with communicating limited border information. In addition, all of the regional optimization models are solved simultaneously and in a parallel manner, instead of a sequential process.

The rest of this paper is organized as follows. Section II presents a co-optimization model for energy and regulation joint dispatch in wind-integrated interconnected power systems. This model is then decomposed per area in Section III and a decentralized algorithm is presented as well. Section IV reports and discusses results obtained from a two-area 6-bus and the modified three-area IEEE 118-bus test systems. Section V concludes the paper.

II. JOINT DISPATCH OF ENERGY AND REGULATION UNDER WIND POWER UNCERTAINTY

This section develops a real-time energy and regulation joint dispatch model of a multi-area interconnected power system. In order to avoid complexity, the entire system is assumed here to be operated as a single area. Then, in Section III, the model is decomposed per area and tackled with a decentralized solution procedure.

The proposed real-time dispatch model is intended for simultaneously dispatching generating units and allocating regulation reserve as well as interregional energy trading in a multi-area power system with high penetration of wind energy. The proposed model is a single-settlement real-time market and is performed in every 5–10 min operating intervals. For each market interval, dispatchable generating units submit their economic data as well as technical limitations. In addition to wind power forecast, predefined intervals (PIs) $[\frac{1}{2}P_{W}, \frac{3}{2}P_{W}]$ are considered to model the wind power variation and uncertainty. The PIs are calculated based on the desired confidence level and lower and upper limits of PIs represent the minimum and maximum generation levels, respectively, corresponding to the confidence interval. Accordingly, wind power variation and uncertainty between two consecutive real-time market intervals are captured via a confidence envelope, as shown in Figure 1. The energy/regulation reserve co-optimization market program

![FIG. 1. Representation of wind power uncertainty.](image-url)
dispatches energy and regulation reserve offers to minimize the total cost of supplying the demand during the real-time operation. The linear programing (LP) model of the problem in question is formulated in (1)–(11)

\[
\text{Min} \sum_{A \in A} \left\{ \sum_{i \in G^A} C_i(P^G_i) + \sum_{i \in G^A} (q_i r_i) \right\},
\]

\[
\sum_{i \in G^A} P^G_i + \sum_{l \in \Gamma^A \cup \Gamma^D} P_l - \sum_{l \in \Gamma^A \cup \Gamma^D} P_l + \sum_{j \in \Gamma^D} P^W_j = P^D \quad \forall a \in N^A,
\]

\[
-F_l \leq P_i = \frac{1}{x_l} (\theta_{pl} - \theta_{pw}) \leq F_l \quad \forall l \in \Gamma^A,
\]

\[
0 \leq r_i \leq v_i F_l \quad \forall i \in G^H
\]

\[
-L_d \cdot RD_l \leq P^G_l - P^G_{\text{lo}} \leq L_d \cdot RU_l \quad \forall i \in G^E,
\]

\[
\begin{cases}
P^G_l + r_l \leq v_i \cdot \frac{P^G}{x_l} \\
\frac{P^G}{x_l} - r_l \geq v_i \cdot \frac{P^G}{x_l} \\
0 \leq r_i \leq v_i F_l \quad \forall i \in G^H
\end{cases}
\]

\[
\sum_{i \in G^A_{lo}} P^G_i + \sum_{i \in G^A_{lo}} P^G_i + \sum_{l \in \Gamma^A \cup \Gamma^D} \tilde{P}_l - \sum_{l \in \Gamma^A \cup \Gamma^D} \tilde{P}_l + \sum_{j \in \Gamma^D} \tilde{P}_j^w = P^D \forall a \in N^A,
\]

\[
-F_l \leq \tilde{P}_l = \frac{1}{x_l} (\hat{\theta}_{pl} - \hat{\theta}_{pw}) \leq F_l \quad \forall l \in \Gamma^A,
\]

\[
-r_i \leq \tilde{P}_i - P^G_{\text{lo}} \leq r_i \quad \forall i \in G^H.
\]

The objective function (1) consists of the cost of individual unit energy and regulation dispatch summed up over the whole areas. Equation (2) represents the nodal power balance constraints. Constraint (3) ensures that the power flow of transmission lines are within their capacity limits. Constraint (4) describes the generation output limits of nonAGC units. Constraint (5) states that the power output plus the regulation reserve of the AGC unit should be within its minimum and maximum generation level. Equation (6) limits the AGC unit regulation reserve. NonAGC and AGC units ramping up and down are limited by (7) and (8), respectively. Constraint (9) guarantees that AGC units can modify their output so that the nodal power balance between two consecutive real-time market intervals are met for all realization of uncertain wind power within PIs. It is worth mentioning that the nodal load uncertainty can readily be added on the right hand side of this constraint as well. Transmission line limits under wind power uncertainty are ensured by Equation (10). Constraint (11) describes the regulation reserve provided by each AGC unit.

In the above optimization problem, the actual power output of AGC units ($\tilde{P}_i^G$) is a dependent uncertain decision variable in terms of uncertain wind power output. This variable is ultimately determined and controlled by the AGC system (Figure 2) to compensate the ACE caused by wind power variation during the real-time operation.

Each AGC unit $i$ in the area $A$ participates in AGC by a participation factor $x_i^A$, while $\sum_{i \in G^A} x_i^A = 1$. The AGC command output of generating unit $i$ ($P_i^c$) is determined by
where $y_A$ is the sum of AGC commands sent to the generators in area $A$, and it can be seen from (12) that $y_A = \sum_{i \in G^A_i} P^C_i$. According to Figure 2, the AGC system dynamic is given by

$$\dot{y}_A = -y_A - ACE_A + \sum_{i \in G^A_i} \tilde{P}^G_i,$$  

where the ACE for the area $A$ is

$$ACE_A = \sum_{l \in Y^i} (\tilde{P}_l - P_l) - B_A(\tilde{f}_A - f^n).$$  

(14)

As the proposed coordination method aims to implement the AGC system in which the entire interconnection is considered as a single area, we use the ACE of combined areas which can be formulated as follows:

$$ACE_X = \sum_{A \in \Lambda} \left( \sum_{l \in Y^A} (\tilde{P}_l - P_l) \right) - \sum_{A \in \Lambda} B_A(\tilde{f}_A - f^n) = -B(\tilde{f} - f^n).$$  

(15)

Since the system is interconnected, it is assumed that the areas are operated under the same frequency ($\tilde{f}_A = \tilde{f}$). In addition, since each balancing area sets its frequency bias factor equal to or close to its actual frequency response characteristic, we can say, that $B = \sum_{A \in \Lambda} B_A$. So, Equation (15) holds. In the proposed model, participating areas collaborate to reduce the accumulated ACE instead of their individual ACEs presented in (14). Accordingly, we modify the ACEs of participating areas, and the ACEs are calculated by distributing the accumulated ACE among them as given in (16).

$$ACE^\text{new}_A = ACE_X \cdot \gamma_A,$$  

(16)

where $ACE^\text{new}_A$ is an adjusted ACE value for area $A$, and $\gamma_A$ is the sharing factor for this area. The $\gamma_A$ factor could be chosen based on different strategies. One way is to select this factor in proportion with the size of the associated area as in Refs. 1 and 22. This strategy is not adaptive to the system situation and does not optimally deploy the regulation resources since it does
not take the economic aspect into account. Here, the sharing factors are adaptively determined based on the participation of each area in the regulation requirement of the entire system. Note that the new regulation sharing factors should satisfy the following condition.

\[ \sum_{A \in A} \gamma_A = 1. \]  \hfill (17)

The objective of the AGC system is to approach the ACE to zero. According to (15), if the frequency deviation becomes zero, i.e., (18) holds, \( A \tilde{E}_\Sigma \) will be zero.

\[ \sum_{A \in A} \sum_{i \in G_A^h} P_{wi}^c = \sum_{A \in A} \sum_{j \in J^A} P_{wj}^w \quad \forall \begin{bmatrix} P_{wi}^w, \tilde{P}_{wj}^w \end{bmatrix}. \]  \hfill (18)

In order to make sure that the ACE goes to zero during the real-time operation, the AGC system should compensate for all possible wind power uncertainty within its PI. In addition, if the ACE converges to zero under the worst-case scenarios, any other realization of wind power would not be problematic. In order to find the worst-case scenarios, the interval linear programming theory\(^\text{24,25}\) is implemented here. Equation (18) is an interval equality constraint in the proposed model, then the worst optimum solution will be obtained at either \( \sum_{A \in A} \sum_{i \in G_A^l} P_{wi}^{cu} = \sum_{A \in A} \sum_{j \in J^A} P_{wj}^w \) or \( \sum_{A \in A} \sum_{i \in G_A^l} P_{wi}^{cd} = \sum_{A \in A} \sum_{j \in J^A} \tilde{P}_{wj}^w \) (See Ref. 24 theorem 6). Accordingly, the lower and upper limits of the wind power are considered as two worst-case scenarios. Holding (18) under these two worst-cases dictates that the nodal power balance (9) must be satisfied by adjusting the output of AGC units for these two worst-case scenarios. That is

\[ \sum_{i \in G_A^{lu}} P_{wi}^{cu} + \sum_{i \in G_A^{lu}} P_{wi}^G + \sum_{l \in \Gamma^A | \eta=(a)} P_{pl}^G - \sum_{l \in \Gamma^A | r=(a)} P_{pl}^w + \sum_{j \notin J^A} \tilde{P}_{wj}^w = P_A \quad \forall a \in N^A, \]  \hfill (19)

\[ \sum_{i \in G_A^{ld}} P_{wi}^{cd} + \sum_{i \in G_A^{ld}} P_{wi}^G + \sum_{l \in \Gamma^A | \eta=(a)} P_{pl}^G - \sum_{l \in \Gamma^A | r=(a)} P_{pl}^w + \sum_{j \notin J^A} \tilde{P}_{wj}^w = P_A \quad \forall a \in N^A, \]  \hfill (20)

where

\[ -\overline{P}_l \leq P_{pl}^w = \frac{1}{x_l} (\theta_{pl}^w - \theta_{pl}^u(l)) \leq \overline{P}_l \quad \forall l \in \Gamma^A, \]  \hfill (21)

\[ -\overline{P}_l \leq P_{pl}^d = \frac{1}{x_l} (\theta_{pl}^d - \theta_{pl}^l(l)) \leq \overline{P}_l \quad \forall l \in \Gamma^A. \]  \hfill (22)

Considering that \( \tilde{P}_{wi}^G \) is ultimately determined and controlled by the AGC system and the output of AGC units should compensate for wind power uncertainty associated with these two worst-case scenarios, the constraint (11) can be rewritten as follows:

\[ P_{wi}^{cu} - P_{wi}^G \leq r_i \quad \forall i \in G_A^H, \]  \hfill (23)

\[ P_{wi}^G - P_{wi}^{cd} \leq r_i \quad \forall i \in G_A^H. \]  \hfill (24)

Therefore, constraints (9)–(11) in the proposed LP model are substituted by (19)–(24) to guarantee load-interchange-generation balance within each area during the real-time operation. Having applied the proposed LP model, the AGC participation factor \( \gamma_i^A \) for AGC unit \( i \) in area \( A \) and sharing factor \( \gamma_A \) associated with area \( A \) are optimally determined by

\[ \gamma_i^A = \sum_{i \in G_A^H} (P_{wi}^{cu} - P_{wi}^{cd}) \],  \hfill (25)
As it is evident from (25) and (26), the participation factors of generating units in the AGC system and areas’ sharing factors are dependent variables and are determined based on the AGC command output under upper and lower limits of PIs, i.e., $P_{Cu}^i$ and $P_{Cd}^i$. Accordingly, the proposed model optimizes the participation factors instead of using predefined classical participation factors (25). In addition, the proposed areas’ sharing factors are optimally calculated to distribute the accumulated ACE among the participating areas (26).

III. DECENTRALIZED MARKET CLEARING FORMULATION

In order to solve the proposed joint dispatch model described in Section II, a central operator with access to all data of the whole system is essential. Such a case is not compatible with the existing practice of operation of multi-area electricity markets. Accordingly, a fully decentralized algorithm based on ALR and ADMM techniques is developed here. In the algorithm, the information privacy and decision independency of regional power systems are preserved, and only limited border information from each area is exchanged.

A. Decomposition strategy

The proposed LP model should be decomposed per area. However, area dispatch problems are mathematically linked together through joint variables, and the optimal solution of one would affect those of others. Indeed, the powers transmitted via tie-lines couple the areas together. As the DC power flow is considered here, the powers transferred through the tie-lines are affected by the phase angles of two ends of the tie-lines (as shown in Figure 3(a)). Recalling (3), (21), and (22), the joint variables of the phase angle of buses correspond to three scenarios, i.e., forecasted, upper, and lower limits of the wind power generation. Hereafter, the scenarios are identified with the superscript $s$ in the phase angle symbols where $S = \{f, u, d\}$ stands for forecasted, upper, and lower limits of the wind power generation, respectively.

In order to fully decompose the proposed model into scalable subproblems, the joint variables associated with each boundary bus (the angles of both terminal buses of the tie-line) are duplicated for each neighboring area as shown in Figure 3(b). Having applied this method, the optimal solution of the whole interconnected system will be achieved if the joint variables have the same values for all areas connected to borderline buses $a$ and $b$. To do so, the bridge variables ($\theta_{a}^{f}, \theta_{b}^{f}$) are introduced. These set of variables guarantee the equality of joint variables among neighboring areas by adding the following area coupling constraints to the dispatch problem of each area.

![FIG. 3. Duplication of joint connection between any two neighboring areas.](image)
Now, the ALR method is adopted to relax the area coupling constraints (27) by adding the first-order and the second-order penalty functions into the objective function (1). The objective function of the corresponding ALR problem is written as

$$\theta^S_{a} = \theta^S_{ac}, \quad \theta^S_{b} = \theta^S_{bc}, \quad (a, b) \in \mathcal{J}C_A.$$ \hspace{1cm} (27)

The ADMM method\textsuperscript{26} is applied to address this issue as discussed in Section III B. The ADMM method\textsuperscript{26} is applied to address this issue as discussed in Section III B. However, the augmented Lagrangian function (28) cannot yet be decomposed due to the product of variables introduced by the second-order penalty function. The ADMM method\textsuperscript{26} is applied to address this issue as discussed in Section III B.

**B. Solution approach**

Based on the ADMM approach, the objective function (28) can be fully decomposed per area by substituting the bridge variables with the average phase angle of boundary buses $a$ and $b$ received from connecting areas. That is

$$\min L = \sum_{A \in \mathcal{A}} \left\{ \sum_{i \in \mathcal{G}_A} C_i \left( P^G_i \right) + \sum_{i \in \mathcal{G}_A} (q_i r_i) \right\} + \sum_{(a,b) \in \mathcal{J}C_A} \sum_{s} \left\{ \lambda^S_a \left( \theta^S_{a} - \overline{\theta}^S_{ac} \right) + \frac{\rho^S}{2} \left\| \theta^S_{a} - \overline{\theta}^S_{ac} \right\|^2 \right\} + \lambda^S_b \left( \theta^S_{b} - \overline{\theta}^S_{bc} \right) + \frac{\rho^S}{2} \left\| \theta^S_{b} - \overline{\theta}^S_{bc} \right\|^2 \right\}, \hspace{1cm} \text{(28)}$$

where $\lambda$ and $\rho$ are the penalty multipliers related to the first-order and the second-order terms, respectively. After relaxing area-coupling constraints, the remaining constraints of the proposed model become decomposable. However, the augmented Lagrangian function (28) cannot yet be decomposed due to the product of variables introduced by the second-order penalty function. The ADMM method\textsuperscript{26} is applied to address this issue as discussed in Section III B.

Based on the ADMM approach, the objective function (28) can be fully decomposed per area by substituting the bridge variables with the average phase angle of boundary buses $a$ and $b$ received from connecting areas. That is

$$\min L_A = \sum_{i \in \mathcal{G}_A} C_i \left( P^G_i \right) + \sum_{i \in \mathcal{G}_A} (q_i r_i) + \sum_{(a,b) \in \mathcal{J}C_A} \sum_{s} \left\{ \lambda^S_a \left( \theta^S_{a} - \overline{\theta}^S_{ac} \right) + \frac{\rho^S}{2} \left\| \theta^S_{a} - \overline{\theta}^S_{ac} \right\|^2 \right\} + \lambda^S_b \left( \theta^S_{b} - \overline{\theta}^S_{bc} \right) + \frac{\rho^S}{2} \left\| \theta^S_{b} - \overline{\theta}^S_{bc} \right\|^2 \right\}, \hspace{1cm} \text{(29)}$$

where $\overline{\theta}^S_{ac}$ and $\overline{\theta}^S_{bc}$ are the average perception of phase angles $\theta^S_{a}$ and $\theta^S_{b}$, by all areas including area $A$ connected to boundary buses $a$ and $b$, respectively. $\overline{\theta}^S_{ac}$ and $\overline{\theta}^S_{bc}$ are calculated as follows:

$$\overline{\theta}^S_{ac} = \frac{\sum_{A' \in \Psi^S_a} \theta^S_{a'}}{|\Psi^S_a|}, \hspace{1cm} \text{(30)}$$

$$\overline{\theta}^S_{bc} = \frac{\sum_{A' \in \Psi^S_b} \theta^S_{b'}}{|\Psi^S_b|}, \hspace{1cm} \text{(31)}$$

where $|\Psi^S_a|$ and $|\Psi^S_b|$ denote the number of areas connected to boundary buses $a$ and $b$, respectively.

Now, the multi-area joint dispatch problem can be iteratively solved in a parallel manner, and the optimal solution of the whole system would be achieved after sufficient iterations. Figure 4 displays the parallel solution procedure based on ADMM. This procedure is implemented as follows.

**Step 1. Initialization:** All areas set the iteration index $k = 1$; choose initial penalty factors $\rho^{(k)}$ as well as $\mu$ and $\tau$; select initial values for the average phase angle of boundary buses $a$ and $b$, $\overline{\theta}^{(k)}_{ac}$ and $\overline{\theta}^{(k)}_{bc}$, and dual variables $\lambda^{(k)}_a$, $\lambda^{(k)}_b$, set the convergence tolerances $\varepsilon^p > 0$ and $\varepsilon^d > 0$. 

$$\overline{\theta}^{(k)}_{ac} \quad \text{and} \quad \overline{\theta}^{(k)}_{bc} \quad \text{are calculated as follows:}$$

$$\overline{\theta}^{(k)}_{ac} = \frac{\sum_{A' \in \Psi^S_a} \theta^S_{a'}}{|\Psi^S_a|}, \hspace{1cm} \text{(30)}$$

$$\overline{\theta}^{(k)}_{bc} = \frac{\sum_{A' \in \Psi^S_b} \theta^S_{b'}}{|\Psi^S_b|}, \hspace{1cm} \text{(31)}$$

where $|\Psi^S_a|$ and $|\Psi^S_b|$ denote the number of areas connected to boundary buses $a$ and $b$, respectively.
Step 2. **Subproblems solving:** Each area $A$ solves its own problem (29), where $\hat{\theta}_a = \hat{\theta}_a^{(k)}$, $\hat{\theta}_b = \hat{\theta}_b^{(k)}$, $\hat{\theta}_{ac} = \theta_{ac}^{(k)}$, and $\hat{\theta}_{bc} = \theta_{bc}^{(k)}$ to obtain the optimal solution.

Step 3. **Information sharing:** Each area $A$ sends the phase angles of boundary buses $\hat{\theta}_a^a$ and $\hat{\theta}_b^a$ to its neighboring areas.

Step 4. **Perception updating:** Each area $A$ updates the average of received phase angle of its boundary buses $\bar{\theta}_a^{(k+1)}$ and $\bar{\theta}_b^{(k+1)}$ using (30) and (31), respectively.

Step 5. **Convergence check:** Each area $A$ calculates primal and dual residues of its area and checks the constraints (32) and (33). If they are met for all the areas, the procedure stops and the converged optimal results is obtained; otherwise, go to Step 6.
Step 6. Dual variables and penalty factors updating: Each area $A$ updates dual variables $\lambda_{S_{a}}^{S(k)}$, $\lambda_{S_{b}}^{S(k)}$, and penalty factors $\rho_{S_{a}}^{S(k)}$ using (34)–(36); sets $k = k + 1$ and goes to Step 2. It should be noted that in order to accelerate the convergence of ADMM in practice as well as making the performance less dependent on the initial choice of penalty factors, an ADMM with variable penalty factors according to what is proposed in Ref. 27 is applied here.

$$
\lambda_{a}^{S(k+1)} = \lambda_{a}^{S(k)} + \rho_{S_{a}}^{S(k)} \left( \theta_{a}^{S(k)} - \theta_{ac}^{S(k+1)} \right), \tag{34}
$$

$$
\lambda_{b}^{S(k+1)} = \lambda_{b}^{S(k)} + \rho_{S_{b}}^{S(k)} \left( \theta_{b}^{S(k)} - \theta_{bc}^{S(k+1)} \right), \tag{35}
$$

$$
\rho_{S_{a}}^{S(k+1)} =
\begin{cases}
\tau \cdot \rho_{S_{a}}^{S(k)} & \text{if } \left( P_{Ra}^{S(k)} > \mu_{Ra} \cdot D_{Ra}^{S(k)} \right) \text{ and } \left( P_{Rb}^{S(k)} > \mu_{Rb} \cdot D_{Rb}^{S(k)} \right), \\
\rho_{S_{a}}^{S(k)} / \tau & \text{if } \left( D_{Ra}^{S(k)} > \mu_{Ra} \cdot P_{Ra}^{S(k)} \right) \text{ and } \left( D_{Rb}^{S(k)} > \mu_{Rb} \cdot P_{Rb}^{S(k)} \right), \\
\rho_{S_{a}}^{S(k)} & \text{otherwise.}
\end{cases} \tag{36}
$$

It should be noted that the proposed model is a convex optimization problem. Accordingly, the solution obtained through the above procedure converges to the global optimal value. A detailed discussion of ADMM convergence can be found in Ref. 26.

IV. CASE STUDIES

The effectiveness of the proposed decentralized multi-area energy and regulation joint dispatch model is evaluated in this section using a two-area 6-bus and the modified three-area IEEE 118-bus test systems. All studies were conducted using CPLEX 12.5.1 and GAMS.29

A. Two-area 6-bus test system

An illustrative case study was carried out on a 6-bus interconnected system. The 6-bus system has 6 generators, 2 wind farms, 4 loads, and 7 lines as shown in Figure 5. The system is composed of two areas $A$ and $B$, which are connected through tie-line B3–B6. All line reactances are 0.25 p.u. (on a base of 100 MW). The capacities of all lines including the tie-lines are equal to 100 MW. Bus 2 is considered as the reference bus. The generators data are given in Figure 5.
Table V in the Appendix. The costs of units in area B are higher than those of units in area A in order to force power transfer from area A to area B. The forecasted production of both wind farms WF1 and WF2 are 52 MW, and their associated forecast error is ±7.8 MW. Demand in each area is 183 MW.

The penalty factors \( \rho^{S(k)} \) of the ADMM procedure are initialized to a value of 2, and parameters for updating penalty factors are \( \tau = 1.3 \) and \( \mu = 10 \). The convergence thresholds are \( \delta^P = 5 \times 10^{-3} \) and \( \delta^D = 10^{-3} \).

To demonstrate effectiveness of the proposed model, the following three cases are derived and discussed in detail:

**Case 1. Coordinated multi-area economic dispatch without regulation sharing:** In this case, both areas A and B clear their energy and regulation markets simultaneously; however, the regulation service for each area is locally optimized.

**Case 2. Coordinated multi-area energy and regulation joint dispatch model:** In this case, balancing resources in both areas A and B are optimally dispatched to provide system-wide energy and regulation requirements.

**Case 3. Coordinated multi-area energy and regulation joint dispatch model with reduced tie-line capacity:** This case is similar to Case 2; however, the tie-line capacity is reduced to half (50 MW).

These cases are tackled using the proposed formulation and the obtained solutions are presented in the following.

Table I compares energy and regulation dispatch associated with both areas A and B in the three cases. In Case 1, since the regulation reserve requirement for each area is locally provided, generating units G1 and G2 in area A and generating units G4 and G5 in area B are dispatched to cope with possible variation and uncertainty of wind farms 1 and 2, respectively. It should also be noted that 71.2 MW of the demand in area B is supplied by generating units in area A via the tie-line.

In Case 2, 3.6 MW (46.15\%) of the required regulation reserve in area B, which is provided by unit G5 in Case 1, is procured by generating units in area A due to low energy and regulation bids of these units in comparison with unit G5 in area B. In addition, area B imports 73.6 MW from area A in this case which is 2.4 MW higher than Case 1.

In Case 3, regulation reserve allocation is similar to Case 2, and 46.4 MW power transfers from area A to area B. This observation shows that although the tie-line capacity is limited to 50 MW (23.6 MW lower than cross border trading in Case 2), 3.6 MW of the tie-line capacity is still allocated for providing regulation service of area B by area A.

Table II compares the system costs including energy, regulation, and total costs of three cases. As it can be seen, the system-wide regulation cost is reduced by 18.4\% in Cases 2 and 3. Moreover, both areas A and B benefit from regulation sharing: area A by selling more expensive regulation reserve and area B by buying cheaper regulation reserve. Note that the costs presented in Table II are calculated based on generation units’ price offer for energy and regulation reserve. With regard to market clearing, the price of energy exchange via the tie-line can be calculated based on the locational marginal prices at two ends of the tie-line. It is interesting to mention that although the capacity of the tie-line is limited in Case 3, its associated total cost is lower than the one obtained from Case 1 in which the cross border trading is not limited.

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Area A</th>
<th>Area B</th>
<th>Area A</th>
<th>Area B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( P_{1}^{c} )</td>
<td>( r_{1} )</td>
<td>( P_{2}^{c} )</td>
<td>( r_{2} )</td>
</tr>
<tr>
<td>Case1</td>
<td>96.4</td>
<td>3.6</td>
<td>95.8</td>
<td>4.2</td>
</tr>
<tr>
<td>Case2</td>
<td>95.8</td>
<td>4.2</td>
<td>95.8</td>
<td>4.2</td>
</tr>
<tr>
<td>Case3</td>
<td>95.8</td>
<td>4.2</td>
<td>68.6</td>
<td>4.2</td>
</tr>
</tbody>
</table>
In order to show the performance of the proposed decentralized procedure in terms of computation time and number of iterations, the tie-line power flows in Case 2 with respect to the number of iteration are depicted in Figure 6. As it can be seen, the proposed procedure converges after 47 iterations when the tie-line power flows under forecasted, upper, and lower limits of the wind power generation are equal for both areas. However, recall that the proposed model is a linear programming one which can be easily solved using available commercial solvers (such as GUROBI, XPRESS, CPLEX, and SCIP) in a very short running time. For example, the average central processing unit (CPU) time for each iteration is 0.127 s, and the total CPU time in Case 2 is about 6 s, which is acceptable considering the operational interval (5–10 min) and the lead-time to market closure (7.5–20 min in US electricity markets).

For the purpose of illustrating the performance of the proposed model in comparison with the existing AGC system during real-time operation, the Matlab/Simulink model of the system is developed as depicted in Figure 7. The parameters of the test system are derived from Ref. 30 and reported in Table VI in the Appendix. It is worth noting that the participation factors of generating units \( \alpha_A \) and sharing factors of areas \( \gamma_A \) are optimally calculated based on Equations (25) and (26), respectively. Thus, these variables are the market clearing output which are sent by the operator.

The following step decreases in the wind power representing lower limit of wind power generation are simultaneously applied to both areas A and B: \( \Delta P_W^A = \Delta P_W^B = -0.078 \text{ pu} \). The frequency deviation and the ACE of each control area resulted from applying the proposed model and the existing coordination model, which are shown in Figures 8 and 9, respectively. It should be noted that, in the existing model, each area employs its own AGC system to maintain interconnection scheduled frequency as well as scheduled power flow between the areas, and no coordination between the areas is present. However, in the proposed model, all

<table>
<thead>
<tr>
<th>Area</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area A</td>
<td>2773.8</td>
<td>2820.6</td>
<td>2412.6</td>
<td>70.8</td>
<td>111.6</td>
<td>111.6</td>
<td>2844.6</td>
<td>2932.2</td>
<td>2524.2</td>
</tr>
<tr>
<td>Area B</td>
<td>1225.6</td>
<td>1158.4</td>
<td>1593.6</td>
<td>138</td>
<td>58.8</td>
<td>58.8</td>
<td>1363.6</td>
<td>1217.2</td>
<td>1652.4</td>
</tr>
<tr>
<td>System-wide</td>
<td>3999.4</td>
<td>3979</td>
<td>4006.2</td>
<td>208.8</td>
<td>170.4</td>
<td>170.4</td>
<td>4208.2</td>
<td>4149.4</td>
<td>4176.6</td>
</tr>
</tbody>
</table>

In order to show the performance of the proposed decentralized procedure in terms of computation time and number of iterations, the tie-line power flows in Case 2 with respect to the number of iteration are depicted in Figure 6. As it can be seen, the proposed procedure converges after 47 iterations when the tie-line power flows under forecasted, upper, and lower limits of the wind power generation are equal for both areas. However, recall that the proposed model is a linear programming one which can be easily solved using available commercial solvers (such as GUROBI, XPRESS, CPLEX, and SCIP) in a very short running time. For example, the average central processing unit (CPU) time for each iteration is 0.127 s, and the total CPU time in Case 2 is about 6 s, which is acceptable considering the operational interval (5–10 min) and the lead-time to market closure (7.5–20 min in US electricity markets).
FIG. 7. Simulation setup of the proposed model for two-area 6-bus test system.

FIG. 8. Frequency deviation under maximum wind disturbance: (a) area A (b) area B.

FIG. 9. Area control error under maximum wind disturbance: (a) area A (b) area B.
participating areas share their regulation resources and collaborate as if the interconnection were composed of a single control area. From the simulation results, it can be observed that the ACE and frequency deviation of two areas get faster back to zero in the proposed model. This is because of omitting the feedback of tie-line power flow deviations in the proposed model. So, there is no attempt to restore these deviations to zero, which results in faster control actions.

In addition, the ACE of area A is much more than the ACE of area B in the proposed model. This is due to higher sharing factor of area A, which results from participation of more economical resources in regulation service. Therefore, the proposed model not only reduces the total regulation procurement cost but also results in lower system-wide ex-post regulation cost and better frequency recovery behaviour.

### B. Three-area modified IEEE 118-bus test system

Here, the three-area modified IEEE 118-bus test system is examined. The system has 54 generating units, 91 loads, and 186 transmission lines (174 internal lines and 12 tie-lines) as given in Ref. 31. The original data are modified to include four wind farms in the test system. Wind farms are at bus 22 in area 1, buses 46 and 68 in area 2, and bus 86 in area 3. Their forecasted generations are 250 MW, 170 MW, 175 MW, and 220 MW, respectively. The online generating units’ cost data along with their regulation reserve capacities are reported in Table VII in the Appendix. The generating units TG10 and TG11 in area 1, TG27, TG28, TG29, and TG44 in area 2, and TG39 and TG43 in area 3 are AGC units. The other generators are nonAGC units.

The penalty factors $q_S(k)$ of the ADMM procedure are initialized to a value of 2, and parameters for updating penalty factors are $\tau = 1.04$ and $\mu = 10$. The convergence thresholds are $\epsilon_P = 0.001$ and $\epsilon_D = 0.005$.

The proposed coordinated multi-area joint dispatch model is applied on the test system and compared with the case in which the regulation service for each area is locally optimized. The results obtained from the model without regulation sharing (Case 1) and the proposed model (Case 2) are presented in Table III. As it can be seen, in the proposed model, 16 MW (42.67%), 51.75 MW (100%), and 14.25 MW (43.18%) of the regulation requirement of area 1, area 2, and area 3 are provided by generating units in area 2, respectively. Also, the system-wide regulation reserve cost is reduced by 13.48% in the proposed model. Interestingly, in the proposed model, total energy cost is also decreased due to reducing the generation of AGC units TG28, TG 29, and TG44 in area 2 to provide more regulation reserve. It should be noted that the number of iterations in Case 2 are 114. Also, the total CPU time associated with this case is about 15 s, which is compatible with existing real-time market practices. Talking about the very large-scale systems with thousands of buses, the computational cumbersome is usually being tackled by parallel processing algorithms which are fortunately applicable in the proposed approach we well.

<table>
<thead>
<tr>
<th>System costs</th>
<th>Area 1</th>
<th>Area 2</th>
<th>Area 3</th>
<th>System-wide</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total generation (MW)</td>
<td>Case 1</td>
<td>615.4</td>
<td>1855.8</td>
<td>446.8</td>
</tr>
<tr>
<td></td>
<td>Case 2</td>
<td>685.9</td>
<td>1702.2</td>
<td>529.9</td>
</tr>
<tr>
<td>Total regulation (MW)</td>
<td>Case 1</td>
<td>37.5</td>
<td>51.75</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>Case 2</td>
<td>21.5</td>
<td>82.0</td>
<td>18.75</td>
</tr>
<tr>
<td>Total energy cost ($)</td>
<td>Case 1</td>
<td>10546.8</td>
<td>24467.1</td>
<td>8115.2</td>
</tr>
<tr>
<td></td>
<td>Case 2</td>
<td>11662.8</td>
<td>21889.6</td>
<td>9414.4</td>
</tr>
<tr>
<td>Total regulation cost ($)</td>
<td>Case 1</td>
<td>453.1</td>
<td>316.5</td>
<td>339.7</td>
</tr>
<tr>
<td></td>
<td>Case 2</td>
<td>218.7</td>
<td>572.4</td>
<td>168.7</td>
</tr>
<tr>
<td>Total operation cost ($)</td>
<td>Case 1</td>
<td>10999.9</td>
<td>24783.6</td>
<td>8454.9</td>
</tr>
<tr>
<td></td>
<td>Case 2</td>
<td>11881.5</td>
<td>22462</td>
<td>9583.1</td>
</tr>
</tbody>
</table>
TABLE IV. Market clearing results under different wind power penetration level.

<table>
<thead>
<tr>
<th>Penetration level (%)</th>
<th>Area 1</th>
<th></th>
<th>Area 2</th>
<th></th>
<th>Area 3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1</td>
<td>Case 2</td>
<td>Case 1</td>
<td>Case 2</td>
<td>Case 1</td>
<td>Case 2</td>
</tr>
<tr>
<td>10</td>
<td>17.22</td>
<td>21.87</td>
<td>27.61</td>
<td>34.12</td>
<td>11.16</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>25.83</td>
<td>21.87</td>
<td>41.42</td>
<td>43.40</td>
<td>16.74</td>
<td>18.75</td>
</tr>
<tr>
<td>20</td>
<td>34.44</td>
<td>21.87</td>
<td>55.23</td>
<td>71.37</td>
<td>22.32</td>
<td>18.75</td>
</tr>
<tr>
<td>25</td>
<td>Inf.</td>
<td>21.87</td>
<td>Inf.</td>
<td>90.00</td>
<td>Inf.</td>
<td>28.113</td>
</tr>
<tr>
<td>27</td>
<td>Inf.</td>
<td>23.69</td>
<td>Inf.</td>
<td>90.00</td>
<td>Inf.</td>
<td>37.50</td>
</tr>
</tbody>
</table>

In order to show the impact of the proposed model on facilitating more wind integration into the power system and coping with its inherent uncertainty, a sensitivity analysis is conducted. To do so, we increase the level of wind power penetration from 10% of each area’s demand by adding 1% wind generation in each step. The results are presented in Table IV. It is observed that the coordinated dispatch model without regulation sharing (Case 1) only allows 20% wind power penetration. For the penetration level beyond 20%, Case 1 becomes infeasible since the participating areas run out of balancing resources to accommodate the possible variability and uncertainty of wind power without threatening the system security and reliability. However, the variation and uncertainty of wind power up to 27% penetration level can be managed with available balancing resources in the proposed model (Case 2). Therefore, the proposed coordination model can be considered as a beneficial strategy to facilitate high-level wind penetration while limiting the requirement for generation reserves.

V. CONCLUSIONS

In this paper, a coordinated multi-area market clearing model was proposed to effectively share regulation reserve and to provide more cost-effective and flexible dispatch pattern in wind integrated interconnected power systems. The proposed coordination model successfully provided the optimal solution that would obtain if all participating areas were under the same jurisdiction, while respecting each region’s information privacy. More specifically, the proposed model used the summation of the individual ACEs to approximate the ACE of combined participating areas and economically distributed it into the AGC system of each area through the proposed adaptive sharing factors. Next, the ALR along with ADMM methods were applied to provide a fully decentralized procedure with limited data communication among neighboring areas. The simulation results demonstrated that the proposed model significantly reduces the overall regulation reserve cost and benefits all participating areas from regulation sharing. In addition, the results showed that more uncertainty associated with integration of wind power into the power grid can be tackled by applying the proposed model. Therefore, the proposed model benefits power system operation from both economic and reliability perspectives. As the applicability of the proposed model is not limited to the real-time electricity market, the proposed coordination methodology can be employed in the future work to address multi-area day-ahead scheduling and planning for variable generation integration issues.

NOMENCLATURE

Sets and indices

\( a \)  Index of buses in area \( A \)
\( A, B \)  Indices of areas
Index of external buses in neighboring areas connected to boundary bus 
Index of bridge variables 
Set of dispatchable units (including AGC and nonAGC units) 
Set of dispatchable units connected to bus 
Set of nonAGC units 
Set of AGC units 
Set of non AGC units connected to bus 
Set of AGC units connected to bus 
Index of dispatchable units 
Index of wind farms 
Set of buses at the both ends of tie-lines 
Set of wind farms connected to bus 
Index of iterations 
Index of transmission lines 
Set of nodes 
Index of wind power generations including forecasted, upper, and lower limits 
Set of lines (including internal and tie-lines) 
Set of areas 
Set of areas (including area A) connected to boundary bus 
Set of tie-lines

Parameters

Frequency bias of area A/whole system [MW/Hz] 
Incremental energy offer cost function [$] 
Damping coefficient [pu/Hz] 
Sending/receiving bus of line 
Scheduled interconnection frequency [Hz] 
Inertia constant [s] 
Dynamic controller 
Interval length [min] 
Maximum capacity of line [MW] 
Power demand [MW] 
Maximum/minimum of \( P_i^G \) [MW] 
Scheduled power from previous interval [MW] 
Wind power forecast as offered in market [MW] 
Predefined interval (PI) of \( P_j^W \) 
Primal and dual residuals 
Price offer for regulation reserve [$/MW] 
Maximum of regulation reserve [MW] 
Droop characteristic [Hz/pu] 
Ramp-down/up limit [MW/min] 
Synchronizing torque coefficient [pu/Hz] 
Speed governor time constant [s] 
Steam turbine time constant [s] 
Binary parameter indicating unit commitment status 
Reactance of line [p.u.] 
Primal and dual tolerances 
Average phase angle of boundary bus a perceived by its connecting areas 
Dual variable corresponding to deviation of actual phase angle from the average perception
\(\mu, \tau\) Parameters for updating penalty factor
\(\rho_{S(k)}\) Penalty factor

**Deterministic variables**

- \(P_l\) Line power flow [MW]
- \(p_{Ci}^{\text{up}} / p_{Ci}^{\text{dn}}\) AGC command output under upper/lower PI [MW]
- \(p_i^G\) Scheduled power [MW]
- \(P_{li}^{\text{up}} / P_{li}^{\text{dn}}\) Line power flow under upper/lower PI [MW]
- \(r_i\) Regulation reserve [MW]
- \(\omega_i^A\) AGC unit participation factor
- \(\gamma_A\) Area’s sharing factor
- \(\theta_{ai}\) Bus phase angle [rad].
- \(\theta_{ai}^{\text{up}} / \theta_{ai}^{\text{dn}}\) Bus phase angle under upper/lower PI [rad]

**Uncertain quantities**

- \(ACE_A\) Raw ACE of area A [MW]
- \(ACE_\Sigma\) Total ACE of all areas [MW]
- \(ACE_{A^{\text{new}}}\) Updated ACE of area A [MW]
- \(f_A\) Possible interconnection frequency at area A [Hz]
- \(P_i\) Possible line power flow [MW]
- \(p_i^C\) AGC unit command output [MW]
- \(\hat{p}_i^G\) Possible power output of AGC unit [MW]
- \(\hat{p}_i^W\) Possible wind farm power output [MW]
- \(\gamma_A\) Sum of AGC commands of area A [MW]
- \(\hat{\theta}_a\) Possible bus phase angle [rad]

**APPENDIX: GENERATOR DATA AND SIMULATION PARAMETERS**

Tables V–VII show the generator data and simulation parameters for the 6-bus and 118-bus test systems.

**TABLE V. Generator data for 6-bus test system.**

<table>
<thead>
<tr>
<th>Unit</th>
<th>Energy ($/MW)</th>
<th>Regulation ($/MW)</th>
<th>(\bar{P}_i^G) (MW)</th>
<th>(\bar{P}_i^C) (MW)</th>
<th>(\tau_i) (MW)</th>
<th>(RU_i), (RD_i) (MW)</th>
<th>(2H_i) (s)</th>
<th>(D_i) (pu/Hz)</th>
<th>(R_i) (pu/Hz)</th>
<th>(T_{gi}) (s)</th>
<th>(T_{ti}) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>12</td>
<td>8</td>
<td>100</td>
<td>10</td>
<td>4.2</td>
<td>0.84</td>
<td>8</td>
<td>0.015</td>
<td>3</td>
<td>0.10</td>
<td>0.45</td>
</tr>
<tr>
<td>G2</td>
<td>15</td>
<td>10</td>
<td>100</td>
<td>10</td>
<td>4.2</td>
<td>0.84</td>
<td>8</td>
<td>0.015</td>
<td>3</td>
<td>0.10</td>
<td>0.45</td>
</tr>
<tr>
<td>G3</td>
<td>18</td>
<td>12</td>
<td>60</td>
<td>10</td>
<td>3</td>
<td>0.6</td>
<td>7</td>
<td>0.015</td>
<td>2.4</td>
<td>0.08</td>
<td>0.40</td>
</tr>
<tr>
<td>G4</td>
<td>16</td>
<td>14</td>
<td>100</td>
<td>10</td>
<td>4.2</td>
<td>0.84</td>
<td>8</td>
<td>0.015</td>
<td>3</td>
<td>0.10</td>
<td>0.45</td>
</tr>
<tr>
<td>G5</td>
<td>24</td>
<td>22</td>
<td>100</td>
<td>10</td>
<td>4.2</td>
<td>0.84</td>
<td>8</td>
<td>0.015</td>
<td>3</td>
<td>0.10</td>
<td>0.45</td>
</tr>
<tr>
<td>G6</td>
<td>32</td>
<td>28</td>
<td>60</td>
<td>10</td>
<td>3</td>
<td>0.6</td>
<td>7</td>
<td>0.015</td>
<td>2.4</td>
<td>0.08</td>
<td>0.40</td>
</tr>
</tbody>
</table>

**TABLE VI. Simulation parameters of 6-bus test system.**

<table>
<thead>
<tr>
<th>Area</th>
<th>K</th>
<th>B</th>
<th>(T_{AB})</th>
<th>(\gamma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.3</td>
<td>1.1283</td>
<td>0.167</td>
<td>0.7308</td>
</tr>
<tr>
<td>B</td>
<td>0.3</td>
<td>1.1283</td>
<td>0.167</td>
<td>0.2692</td>
</tr>
<tr>
<td>Unit</td>
<td>Energy ($/MW)</td>
<td>Regulation ($/MW)</td>
<td>( \pi ) (MW)</td>
<td>Unit</td>
</tr>
<tr>
<td>-------</td>
<td>---------------</td>
<td>-------------------</td>
<td>----------------</td>
<td>-------</td>
</tr>
<tr>
<td>TG4</td>
<td>18</td>
<td>...</td>
<td>...</td>
<td>TG21</td>
</tr>
<tr>
<td>TG5</td>
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<td>...</td>
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<td>TG24</td>
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<td>TG10</td>
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<td>15</td>
<td>18.75</td>
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</tr>
<tr>
<td>TG11</td>
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<td>21.88</td>
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</tr>
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<td>TG20</td>
<td>18</td>
<td>...</td>
<td>...</td>
<td>TG29</td>
</tr>
</tbody>
</table>

32. See http://motor.ece.iit.edu/data/IEAS_IEEE118 for The IEEE 118-bus test system data.