Coupled Finite-Element/Boundary-Element Analysis of a Reciprocating Self-Excited Induction Generator in a Harmonic Domain

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This paper suggests a general method for analysis of a reciprocating self-excited induction generator based on the coupled finite-element/boundary-element method in a harmonic domain. The finite-element method is used for iron and copper parts in order to deal with nonlinearity and eddy currents, while the boundary-element method is utilized for the air-gap region between the moving parts using a free-space Green function that facilitates the application of a linear time periodic movement. The proposed method leads to a static global matrix that is symmetrical for particular boundary conditions. The results agree well with those obtained by the time-stepping methods.

***Index Terms*—Coupled finite-element/boundary-element method, harmonic balance method, reciprocating self-excited induction generator.**

I. Introduction

INDUCTION generators are inexpensive and have no separate excitation; they can operate for a long period with no particular maintenance. Therefore, self-excited induction generators (SEIGs) can supply electrical power of an area far from the power system transmission lines and where nonconventional energies such as wind energy are available. However, SEIGs have their own disadvantages, including large dependency of the output voltage on the generator speed and load and the stator terminal capacitance requirement, which requires accounting for the speed and load variations. The models that are used for analysis of the SEIG are classified into two major groups. In the first group, a per phase equivalent circuit is utilized; in this case, the nodal-admittance method or loop-impedance method is used to establish the relationships of the machine-related parameters such as load, speed, and capacitance [1]. The second group uses the dq model; other equations, expressing the dependency between the steady-state parameters of the machine, are obtained using the harmonic balance method [2], [3]. In the above-mentioned methods, an attempt has been made to solve a nonlinear equation by an iterative procedure, where the experimental magnetization inductance versus magnetization current are available.

In this paper, a reciprocating SEIG with tubular structure, as shown in Fig. 1, is proposed. The SEIG is applicable in a free-piston generator system that is a combination of a linear engine and linear alternator. It is used in hybrid vehicles because this combination is compact, light, and very reliable [4], [5]. It has d and q windings, shunt exciting capacitances (C_q, C_d), and resistive loads. Its model is nonlinear because of magnetic saturation, longitudinal end-effect, and unbalanced winding distribution that make the model difficult to deal with by analytical methods; thus, numerical methods are employed for its analysis. A transverse edge effect does not exist due to the cylindrical construction.

This paper uses the coupled finite-element/boundary-element (FE–BE) method in harmonic domain for modeling a reciprocating SEIG [6]. Iron and copper parts of the generator are modeled by the FE method; therefore, the nonlinearity and eddy currents are taken into account. The air-gap region between the moving parts is modeled by the BE method using the free-space Green function. This method is especially suitable when a linear motion is involved in the electromagnetic devices to uncouple the moving and the stationary meshes. Here, the method is extended to the time periodic movement. For the particular case of a two-dimensional (2-D) coupled FE–BE model of a linear machine, it only requires elementary manipulations of the Green function and its normal derivative over a fundamental period. In this manner, the SEIG model equations using time-stepping numerical methods is converted to the matrix form of the static coupled FE–BE equations, where they are used to obtain the state variables of the steady-state operation.

The proposed method leads to a static global matrix that is symmetrical for particular boundary conditions. However, this symmetry does not hold, in general, for the BE method. The results agree well with those obtained by time-stepping coupled FE–BE and FE methods.

II. Coupled FE–BE Method in Harmonic Domain With Movement

A. FE Regions

The primary and secondary domains were meshed using three-node triangular finite elements as shown in Fig. 2. The

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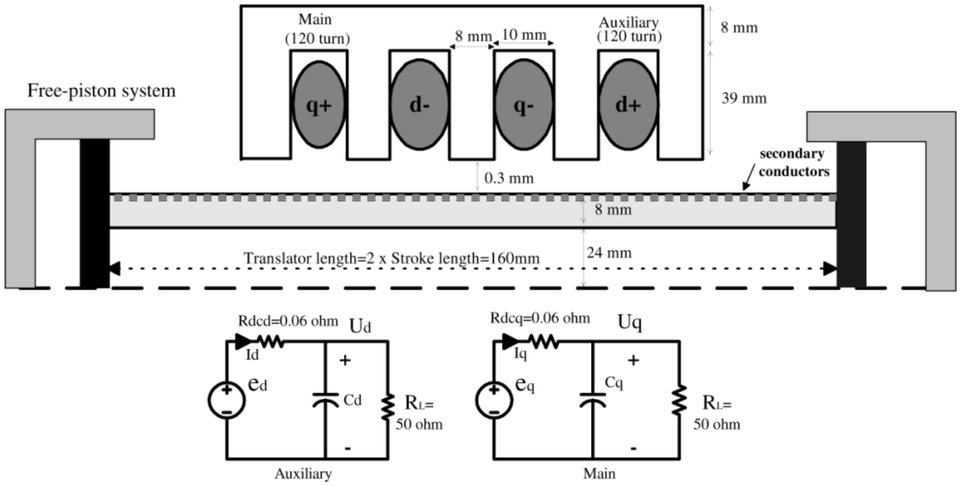

Fig. 1. Reciprocating SEIG and equivalent circuits of d and q windings.

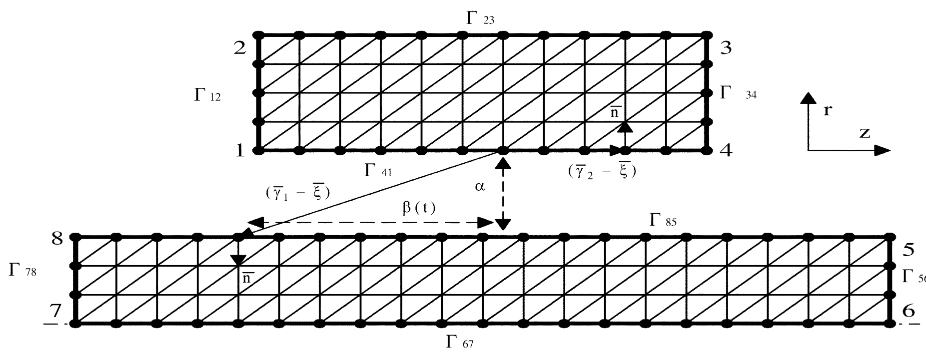

Fig. 2. Mesh of system.

equations are obtained using the Galerkin method. If F denotes the finite element region, based on the equivalent circuits of Fig. 1, the coupled field-circuit matrix equation of the proposed reciprocating SEIG (RSEIG) is as follows [7]–[9]:

$$\begin{bmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} & \mathbf{C}_{13} & \mathbf{C}_{14} \\ \mathbf{0} & \mathbf{C}_{22} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{C}_{33} & \mathbf{0} \end{bmatrix} \begin{bmatrix} A'_F \\ e \\ u \\ \frac{\upsilon_0}{r} A'^n_F \end{bmatrix} + \begin{bmatrix} \mathbf{D}_{11} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{C}^{\mathbf{T}}_{\mathbf{12}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{C}^{\mathbf{T}}_{\mathbf{13}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} A'_F \\ e \\ u \\ \frac{\upsilon_0}{r} A'^n_F \end{bmatrix} = 0. \quad (1)$$

$A_F(A'_F = rA_F)$ is the magnetic potential and $[A'^n_F]_{\Gamma_e} = (\partial A'_F/\partial n)$ is its normal derivative for those elements which have a side shared with the exterior domain Γ. The exterior domain is broadened to lie outside the magnetic iron core and υ_0 is the reluctivity of the air. $e = [e_q e_d]^T$ and $u = [u_q u_d]^T$ are the electromotive forces and terminal voltages of the windings q and d, respectively.

$[\mathbf{C}_{11}]$ is the stiffness sparse symmetrical matrix, $[\mathbf{D}_{11}]$ is the damping symmetrical matrix due to eddy currents, $[\mathbf{C}_{12}]$ and $[\mathbf{C}_{13}]$ are the matrices for taking into account the forcing current densities J_q and J_d in the primary windings, as shown in Fig. 1, $([\mathbf{C}_{12}] \cdot e - [\mathbf{C}_{13}] \cdot u = [J])$. $[\mathbf{C}_{14}]$ takes the exterior domain into account, and e is obtained as differential expressions of $[A'_F]$ using matrices $[\mathbf{C}_{22}]$ and $[\mathbf{C}^{\mathbf{T}}_{12}]$. Circuit equations are incorporated using matrices $[\mathbf{C}_{33}]$ and $[\mathbf{C}^{\mathbf{T}}_{13}]$ with a nodal method [8].

B. BE Equations

The BE method is applied to the air gap that connects the primary and secondary domains [7], [8], [10]. This linear region with constant magnetic permeability $\mu_0(=1/\upsilon_0)$ is geometrically approximated by three-node boundary elements extracted from the primary and secondary element meshes. The contribution of all boundary elements to the magnetic field at a load point $\bar{\xi} = (r_s, z_s)_{\%}$, in terms of the magnetic vector potential, is given by

$$C(\xi)A'_B(\xi) + \sum_{e=1}^{E}\left(\sum_{m=1}^{M} A'^e_{mB} \int_{\Gamma_\gamma} \frac{\partial G(\gamma,\xi)}{\partial n} \phi_m d\Gamma_\gamma\right) = \sum_{e=1}^{E}\left(\sum_{m=1}^{M} \frac{\partial A'^e_{mB}}{\partial n} \int_{\Gamma_\gamma} G(\gamma,\xi)\phi_m d\Gamma_\gamma\right) \quad (2)$$

$$A'^e_B(\xi) = \sum_{m=1}^{M} A'^e_{mB}\phi_m(\xi) \quad (3)$$

$$\frac{\partial A'^e_B(\xi)}{\partial n} = \sum_{m=1}^{M} \frac{\partial A'^e_{mB}}{\partial n}\phi_m(\xi). \quad (4)$$

A'^e_{mB} and $\partial A'^e_{mB}/\partial n$ are the nodal values of magnetic vector potential and its normal derivative, respectively. $C(\xi)$ is the boundary factor and ϕ_m is the linear shape function between each of the two adjacent boundary elements. G is the Green function in axisymmetric coordinates

$$G(\gamma,\xi)=\frac{1}{\pi k}\sqrt{rr_s}\times\left[\left(1-\frac{k^2}{2}\right)K(k)-E(k)\right]$$
$$k^2=\frac{4rr_s}{(r+r_s)^2+(z-z_s)^2},\quad (\bar{\xi}=(r_s,z_s),\bar{\gamma}=(r,z)). \tag{5}$$

E and K are complete elliptic integrals of the first and second kind, respectively, and the derivative $(\partial G/\partial n)$ is either $(\partial G/\partial z)$ or $(\partial G/\partial r)+G/r$, according to the observation point [11].

The application of this formulation to all boundary nodes, using a collocation method, generates a BE system of equations. The values of the magnetic vector potential and its normal derivative on the nodes are unknowns

$$\begin{bmatrix} S_{ppB} & S_{psB}(t) \\ S_{spB}(t) & S_{ssB}\end{bmatrix}\begin{bmatrix} A'_{pB} \\ A'_{sB}\end{bmatrix} + \begin{bmatrix} N_{ppB} & N_{psB}(t) \\ N_{spB}(t) & N_{ssB}\end{bmatrix}\begin{bmatrix} \frac{v_0}{r}A'^n_{pB} \\ \frac{v_0}{r}A'^n_{ssB}\end{bmatrix}=0. \tag{6}$$

The general expressions for S and N are as follows:

$$S_{ij}=\begin{cases}\sum_{m=1}^{M}\int_{\Gamma_{\gamma_j}}\frac{\partial G(\gamma_j,\xi_i)}{\partial n}\phi_m(\xi_i)d\Gamma_{\gamma_j}, & \text{if } i\neq j\\ C(\xi_i)+\sum_{m=1}^{M}\int_{\Gamma_{\gamma_j}}\frac{\partial G(\gamma_j,\xi_i)}{\partial n}\phi_m(\xi_i)d\Gamma_{\gamma_j}, & \text{if } i=j\end{cases} \tag{7}$$

$$n_{ij}=\sum_{m=1}^{M}\int_{\Gamma_{\gamma_j}}G(\gamma_j,\xi_i)\phi_m d\Gamma_{\gamma_j} \tag{8}$$

where A^n_{pB} and A^n_{SB} denote the normal derivatives and $S_{psB}(t)$ and $S_{SpB}(t)$ correspond to the mutual effects of the boundary elements of primary (p) and secondary (s), when one of the load or field points is placed on the primary boundary and the other one is placed on the secondary boundary. Movement causes $S_{psB}(t)$ and $S_{SpB}(t)$ to be time variable as the distance $|\bar{\gamma}-\bar{\xi}|$ varies with displacement. Similarly, the same holds for $n_{psB}(t)$ and $n_{SpB}(t)$.

A combination of (1) and (6) gives the following coupled equations of the model:

$$\begin{bmatrix}\mathbf{C}_{11} & \mathbf{C}_{12} & \mathbf{C}_{13} & \mathbf{C}_{14}\\ \mathbf{0} & \mathbf{C}_{22} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{C}_{33} & \mathbf{0}\\ \mathbf{S} & \mathbf{0} & \mathbf{0} & \mathbf{N}\end{bmatrix}\begin{bmatrix}A'\\ e\\ u\\ \frac{v_0}{r}A'^n\end{bmatrix} + \begin{bmatrix}\mathbf{D}_{11} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{C_{12}^T} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{C_{13}^T} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}\frac{d}{dt}\begin{bmatrix}A'\\ e\\ U\\ \frac{v_0}{r}A'^n\end{bmatrix}=0 \tag{9}$$

where the continuity of the quantities A' and A'^n has been considered

$$\left[\frac{v_0}{r}A'^n_F\right]=-\left[\frac{v_0}{r}A'^n_B\right]. \tag{10}$$

C. Harmonic Balance

The periodic time variation of variables is approximated by a truncated Fourier series with frequency f and period $t=1/F$, where f is the fundamental frequency of the reciprocating motion. Considering h as nonzero harmonics, the corresponding $2H+1$ time-basis functions $H(t)$ are

$$H_0(t)=1,\quad H_{2\lambda-1}(t)=\sqrt{2}\cos(2\pi\lambda ft)$$
$$H_{2\lambda}(t)=-\sqrt{2}\sin(2\pi\lambda ft)\quad (0<\lambda\leq H). \tag{11}$$

The harmonic time discretization of $A'(t)$, $A'^n_B(t)$, $e(t)$ and $u(t)$ can thus be written as [12]

$$A'(t)=\sum_{\lambda=1}^{H}A'^{(\lambda)}H_\lambda(t),\quad A'^n_B(t)=\sum_{\lambda=1}^{H}A'^{n(\lambda)}_B H_\lambda(t)$$
$$e(t)=\sum_{\lambda=1}^{H}e^{(\lambda)}H_\lambda(t),\quad U(t)=\sum_{\lambda=1}^{h}u^{(\lambda)}H_\lambda(t). \tag{12}$$

The harmonic balance (HB) system of algebraic equations can be obtained by using the harmonic basis functions as weighting functions as well. Considering system of (7) results in

$$\frac{1}{T}\int_0^T\left(\begin{bmatrix}\mathbf{C_{11}} & \mathbf{C_{12}} & \mathbf{C_{13}} & \mathbf{C_{14}}\\ \mathbf{0} & \mathbf{C_{22}} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{C_{33}} & \mathbf{0}\\ S & \mathbf{0} & \mathbf{0} & \mathbf{N}\end{bmatrix}\begin{bmatrix}A'\\ e\\ u\\ \frac{v_0}{r}A'^n\end{bmatrix} + \begin{bmatrix}\mathbf{D_{11}} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{C_{12}^T} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{C_{13}^T} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}\frac{d}{dt}\begin{bmatrix}A'\\ e\\ u\\ \frac{v_0}{r}A'^n\end{bmatrix}\right)H_k(t)dt=0. \tag{13}$$

The application of the time discretization leads to the following system of equations:

$$\begin{bmatrix}\mathbf{C_{11H}+D_{11H}} & \mathbf{C_{12H}} & \mathbf{C_{13H}} & \mathbf{C_{14H}}\\ \mathbf{C_{12H}^T} & \mathbf{C}_{22\mathbf{H}} & \mathbf{0} & \mathbf{0}\\ \mathbf{C_{13H}^T} & \mathbf{0} & \mathbf{C_{33H}} & \mathbf{0}\\ \mathbf{S_H} & \mathbf{0} & \mathbf{0} & \mathbf{N_H}\end{bmatrix}\begin{bmatrix}A'_H\\ e_H\\ u_H\\ \frac{v_0}{r}A'^n_H\end{bmatrix}=0 \tag{14}$$

where $[A'^t_H\ e^t_H\ u^t_H\ \frac{v_0}{r}\ A'^{n^t}_H]^t$ are the vectors of harmonic coefficients. If (k,λ) denotes a pair of harmonic functions, then

$$D^{(k,\lambda)}_{11H}=\frac{1}{T}\int_0^T D_{11}H_\lambda(t)H_k(t)dt=D_{11}\delta_{k,\lambda} \tag{15}$$
$$C^{(k,\lambda)}_{12H}=\frac{1}{T}\int_0^T C_{12}H_\lambda(t)H_k(t)dt=C_{12}\delta_{k,\lambda} \tag{16}$$
$$C^{(k,\lambda)}_{13H}=\frac{1}{T}\int_0^T C_{13}H_\lambda(t)H_k(t)dt=C_{13}\delta_{k,\lambda} \tag{17}$$
$$C^{(k,\lambda)}_{14H}=\frac{1}{T}\int_0^T C_{14}H_\lambda(t)H_k(t)dt=C_{14}\delta_{k,\lambda} \tag{18}$$
$$C^{(k,\lambda)}_{22H}=\frac{1}{T}\int_0^T C_{22}H_\lambda(t)H_k(t)dt=C_{22}\delta_{k,\lambda} \tag{19}$$
$$C^{(k,\lambda)}_{33H}=\frac{1}{T}\int_0^T C_{33}H_\lambda(t)H_k(t)dt=C_{33}\delta_{k,\lambda} \tag{20}$$
$$S^{(k,\lambda)}_{ppBH}=1T\int_0^T S_{ppB}H_\lambda(t)H_k(t)dt=S_{ppB}\delta_{k,\lambda} \tag{21}$$
$$N^{(k,\lambda)}_{ppBH}=\frac{1}{T}\int_0^T N_{ppB}H_\lambda(t)H_k(t)dt=N_{ppB}\delta_{k,\lambda} \tag{22}$$