HERMITE-HADAMARD TYPE INEQUALITIES FOR THE
PRODUCT TWO MAPPINGS WHOSE DERIVATIVES
ABSOLUTE VALUES ARE $s$-CONVEX

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ABSTRACT. In this paper, we extend some estimates of the right hand side of a
Hermite-Hadamard type inequality for the product two differentiable functions
whose derivatives absolute values are $s$-convex. Some natural applications to
special weighted means of real numbers are given. Finally, an error estimate
for the Simpson’s formula is also addressed.

1. Introduction

Let $f : [c,d] \subset \mathbb{R} \to \mathbb{R}$ be a convex function and $a,b \in [c,d], a < b$. We consider
the well-known Hadamard’s inequality

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a} \int_{a}^{b} f(x)dx \leq \frac{f(a) + f(b)}{2}.$$  

Both inequalities hold in the reversed direction if $f$ is concave. The Hermite-
Hadamard’s inequality may be regarded as a refinement of the concept of convexity
and it follows easily from Jensen’s inequality. This inequality has several applications in
nonlinear analysis and the geometry of Banach spaces, see [8]. In the recent
years several extensions and generalizations have been considered for classical con-
vexity. We would like to refer the reader to [1, 3, 5, 9, 12] and references therein for
more information. A number of papers have been written on this inequality providing
some inequalities analogous to Hadamard’s inequality given in (1.1) involving
two convex functions, see [2, 10, 11].

The classical Hermite-Hadamard inequality provides estimates of the mean value
of a continuous convex function $f : [a,b] \to \mathbb{R}$.

For functions $f : [a,b] \to \mathbb{R}$ that are differentiable on $(a,b)$, Dragomir and
Agarwal [4] used the formula,