EXTENSIONS OF SAEIDI’S PROPOSITIONS FOR FINDING A UNIQUE SOLUTION OF A VARIATIONAL INEQUALITY FOR \((u,v)\)-COCOERCIVE MAPPINGS IN BANACH SPACES

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Abstract. Let \(C\) be a nonempty closed convex subset of a real Banach space \(E\), let \(B: C \rightarrow E\) be a nonlinear map, and let \(u, v\) be positive numbers. In this paper, we show that the generalized variational inequality \(VI(C,B)\) is singleton for \((u,v)\)-cocoercive mappings under appropriate assumptions on Banach spaces. The main results are extensions of the Saeidi’s Propositions for finding a unique solution of the variational inequality for \((u,v)\)-cocoercive mappings in Banach spaces.

1. Introduction

Let \(C\) be a nonempty closed convex subset of a real normed linear space \(E\) and \(E^*\) be the dual space of \(E\). Suppose that \(\langle \, \cdot \, , \cdot \, \rangle\) denote the pairing between \(E\) and \(E^*\). The normalized duality mapping \(J: E \rightarrow E^*\) is defined by
\[
J(x) = \{ f \in E^* : \langle x, f \rangle = \|x\|^2 = \|f\|^2 \}
\]
for each \(x \in E\). Suppose that \(U = \{ x \in E : \|x\| = 1 \}\). A Banach space \(E\) is called smooth if for all \(x \in U\), there exists a unique functional \(j_x \in E^*\) such that \(\langle x, j_x \rangle = \|x\|\) and \(\|j_x\| = 1\) (see [1]).

Recall the following definitions and examples:

(i) Let \(C\) be a nonempty closed convex subset of a real normed linear space \(E\). A mapping \(T\) of \(C\) into itself is said to be

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