Quantum Genetic Learning Control of Quantum Ensembles with Hamiltonian Uncertainties

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Abstract: In this paper, a new method for controlling a quantum ensemble that its members have uncertainties in Hamiltonian parameters is designed. Based on combining the sampling-based learning control (SLC) and a new quantum genetic algorithm (QGA) method, the control of an ensemble of a two-level quantum system with Hamiltonian uncertainties is achieved. To simultaneously transfer the ensemble members to a desired state, an SLC algorithm is designed. For reducing the transfer error significantly, an optimization problem is defined. Considering the advantages of QGA and the nature of the problem, the optimization problem by using the QGA method is solved. For this purpose, \( N \) samples through sampling of the uncertainty parameters via uniform distribution are generated and an augmented system is also created. By using QGA in the training step, the best control signal is obtained. To test the performance and validation of the method, the obtained control is implemented for some random selected samples. A couple of examples are simulated for investigating the proposed model. The results of the simulations indicate the effectiveness and the advantages of the proposed method.

Keywords: quantum control; quantum genetic algorithm; sampling-based learning control (SLC)

1. Introduction

In quantum phenomena, as in the classical systems, the existence of uncertainties and noises are unavoidable. For example, in superconducting qubits, the coupling energy of a Josephson junction may have fluctuations [1,2]. Noises and fluctuations may exist in magnetic fields and electric fields in cavity quantum electrodynamics (QED) [3,4]. The spins of an ensemble in nuclear magnetic resonance (NMR) experiments may not be exactly known with respect to the strength of the applied radio frequency field [5].

The classification of inhomogeneous quantum ensembles is a significant issue which has many applications in the discrimination of atoms (or molecules), the separation of isotopic molecules, and quantum information extraction. Thus, treating the quantum systems with uncertainties is an important and applicable subject which needs to be considered.

A quantum ensemble consists of a large number of single quantum systems. In the practical world, some of the quantum systems exist in the form of quantum ensembles. Each single quantum system in a quantum ensemble is referred to as a member of the ensemble [6]. Quantum ensembles have wide applications in emerging quantum technology, including long-distance quantum communication [7], quantum computation [8], and magnetic resonance imaging [9].

Control of inhomogeneous quantum ensembles is an important issue in practical applications. Control of inhomogeneous quantum systems for discrimination between two or more similar systems, for instance, is an attractive field of study [10]. In practical applications, the members of quantum ensembles could have variations in some parameters of dynamic systems. These situations are referred to as inhomogeneous quantum ensembles [6].
There are many approaches which can be used for solving quantum control problems with uncertainties. For instance, an optimal control for NMR pulse sequences is designed by applying gradient algorithms [11]. Additionally, a sequential convex programming method is proposed for designing robust quantum manipulations [12]. Dong and his colleagues have designed a development of the variable structure control approach with sliding modes to improve the robustness of quantum systems in which a sliding mode control method is presented for two-level quantum systems to treat bounded uncertainties in the system Hamiltonian [13]. In addition to these works, a Lyapunov control method is presented to attain a universal quantum control [14]. For the first time a sampling-based learning control (SLC) of inhomogeneous quantum ensembles is presented for overcoming the compensation for parameter dispersion [6]. As an important application, the sampling-based learning controller is used for designing of a superconducting quantum control of systems [15]. Construction of universal quantum gates by using a sampling-based learning control are presented in order to find robust optimal control fields in the presence of different fluctuations and uncertainties [16]. Furthermore, an extended sampling-based learning control for designing a robust quantum unitary transformation in quantum information processing is presented and implemented [17]. In other applications, to prevent a control field failing in laser-assisted collisions, a sampling-based robust control is used [18].

In [19], a systematic sampling-based learning control method with gradient-based learning algorithms for steering the components of inhomogeneous quantum ensembles with uncertainties to the same ideal state is investigated by Dong and coworkers. There are some challenges in gradient algorithms. For instance, they may fall into a local optimum depending on the initial choices of problem variables or, in complex situations, function derivatives may not be easily found.

Genetic-type algorithms (GAs) have been used in optimization problem-solving. For this purpose, by applying cross-over and mutation operators on current solutions, new solutions are generated and, statistically, they are moving toward optimal solutions in the search space. The set of solutions, however, converges to an optimum solution according to the principle of the Darwinian theory of evolution.

The quantum genetic algorithm (QGA) was identified by Narayanan and Moore [20]. The QGA, with even a smaller population, presents a great ability of global optimization and good robustness. Therefore, as compared with the common genetic algorithm, QGA has greater effectiveness [21,22]. QGAs are mostly constructed based on qubits (or quantum bits) and state superposition in quantum mechanics. In contrast to classical representations of chromosomes (a binary string, for instance), here they are represented by vectors of qubits (quantum registers).

In this paper, for controlling the quantum systems with uncertainties, a hybrid method based on the SLC method and QGA is used. Specially, artificial samples are generated by sampling the uncertainty parameters in the system model and an augmented system is constructed by using these samples in the training step. Then, to train a control law with the desired performance for the augmented system, QG (quantum genetic) learning and optimization algorithms are used. In the process of testing, a set of selected uncertainty samples is tested to evaluate the control performance. Additionally, an improvement of QGA is conducted to attain better results. In [22] an adding quantum mutation operation in the conventional quantum genetic algorithm is used as an improving device. Quantum mutation, by swapping the value of the probability amplitude of qubits (α, β), can completely reverse the individual’s evolutionary direction. In this paper the mutation operation is implemented on measured qubits (bit strings), which is more effective than adding quantum mutation. Reduction of learning iterations, test error and training error, and also increasing the fidelity index are advantages of the proposed method.

This paper is organized as follows: Section 2 represents the quantum control model and formulates the control problem; A quantum genetic learning ensemble control algorithm is designed in Section 3; Simulation results and control performance are illustrated in Section 4; Conclusions are presented in Section 5.