Switching optimal adaptive trajectory tracking control of quantum systems

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Summary
In this paper, a switching optimal adaptive controller for tracking a time-dependent trajectory in finite-dimensional closed quantum systems is proposed. The issue of intrinsic singularities in trajectory tracking control of quantum systems leads to a sharp rise in the control amplitude. To overcome this drawback, a switching optimal adaptive quantum controller is designed based on Lyapunov stability theory and optimal quantum control approach. A state-dependent strategy is considered to select the switching signal. The new switching controller adjusts the quantum state so that its population traces the desired dynamic trajectory and simultaneously eliminates the effects of singularities and reduces the control amplitude. The proposed controller is tested successfully for population transfer in a 4-level closed quantum system in a simulation experiment. Both issues of reduction of the tracking error and control intensity along with a significant decrease in the number of singular points are well illustrated by simulation experiment as the advantages of the proposed method.

KEYWORDS
adaptive trajectory tracking, optimal quantum control, quantum systems, switching control, tracking control

1 | INTRODUCTION

Nowadays, quantum control theory is extensively developing as one of scientific research areas; also, the development in laser technology is an incentive for quantum control extension. Control of quantum phenomena has been employed in many application systems, such as quantum chemistry, biological processes, population control, quantum information, quantum optics, nanostructure semiconductors, and nuclear magnetic resonance. The main objective of quantum control is manipulating the quantum systems effectively to drive the control system under consideration from an arbitrary initial state into the desired target state. During recent decades, many traditional control methods include the Lyapunov control, optimal control, robust control, and adaptive control have been developed to control the quantum systems. From the perspective of control theory, when the target state of the control system under consideration is a time-dependent function, the control problem is considered as a trajectory tracking problem. Despite the complexity of quantum tracking control, which occurs due to several kinds of quantum states include entangled states, mixed states, and superposition states, several approaches have been developed for trajectory tracking in quantum control systems. For example, Lyapunov trajectory tracking control methods are taken into account to adjust the state of the quantum...
system with Liouville equation, the orbit tracking of a free evolutionary target system in closed quantum systems is considered in the work of Cong and Liu, reference trajectory tracking of coherent quantum systems is developed by Mirrahimi et al, and trajectory tracking of quantum systems with Schrödinger equation is studied in the works of Coron et al and Mirrahimi and Rouchon. In trajectory tracking control of quantum systems, singularities may occur in control laws. These singularities are classified as intrinsic and removable according to their controllability characteristics. The quantum control system passes through the removable singularity, commonly via large amplitude control signals, and still follows the target trajectory. However, the target trajectory cannot be followed by the control system while an intrinsic singularity occurs. To resolve these obstacles, Zhu et al have introduced a rank index, which characterizes nontrivial singularities in control laws and resolves the singularities by a proper limiting process. Besides, the adaptive trajectory tracking (ATT) control has been utilized to improve the stability of quantum control systems, which comprise the singularities and large amplitude control signals.

On the other hand, the classical optimal control theory provides powerful tools and concepts, which can be exploited in quantum control systems. The optimal control methods have been used as formal approaches to solve the problem of steering a desired initial quantum state to the set of given final states or one given target state. In particular, the optimal control can manipulate the quantum systems to achieve the control objective while additional features of the transition process, such as power expenditure, the energy function, the transfer time, and the fidelity are optimized. Accordingly, many studies have published on developing new optimal control techniques for quantum control systems. An optimal population transfer in the limit of large transfer time, time optimal control of quantum systems, a study on critical points of the optimal quantum control, Pareto optimal quantum control for simultaneous control of an arbitrary number of quantum observable expectation values, optimal control for quantum systems despite the feedback delay, robust optimal control of quantum systems in the presence of the disturbances and the uncertainties, numerical cascade nonlinear conjugate gradient scheme, and nonlinear conjugate gradient scheme. Moreover, several numerical methods have been presented to solve the optimization problem in optimal quantum control including a symmetric split operator method, a semidiscrete paradigm composed of finite element method and conjugate gradient method, the pseudospectral method, a gradient descent algorithm equipped with adaptive step size selection, and greedy algorithms. During recent years, optimal quantum control has been applied in many fields, such as quantum chemistry, quantum computation, quantum information, quantum mechanics, and quantum optics.

In some control systems, finding a continuous static or dynamic feedback law that solves the control problem is impractical. In such situations, an alternative is to incorporate logic-based decisions in the control law and implement the switching between a family of controllers. The switching control has primarily motivated by the problem of designing a controller so that the control system exhibits the desired performance. During last decades, traditional switching control has been of great interest in engineering systems including mechanical systems, automotive industry, aircraft traffic control, switching power converters, and many other fields. For example, we can mention resilient estimator design for an automotive suspension control system, discrete-time switching with redundant channels in observer-based control for communication networks, adaptive fault-tolerant control based on a supervisory switching strategy for actuator failures. Switching controllers have been applied in unstable systems to improve the stability and control performance. Recently, switching control has been extended to quantum control systems. It has been utilized to provide a robust strategy for preparing the arbitrary pure states of an incoherent qubit system. Since controllability is a very important issue in quantum control theory, Dong and Petersen have used the switching control to guarantee the wavefunction controllability of a class of quantum systems. Furthermore, a switching control strategy has considered achieving the controllability in state transfer of closed quantum systems under a degenerate case.

In this paper, by combining ATT control and optimal quantum control, we introduce a new switching approach for trajectory tracking control in a finite-dimensional closed quantum system. The initial state of the quantum system is considered arbitrary, and its target state is a time-dependent function. The adaptive laws are designed based on Lyapunov stability theory. Then, we consider the quantum control system as a switched system and select the switching signal based on a state-dependent strategy. Subsequently, the intrinsic singularity and large control amplitude drawbacks are eliminated by activating the optimal quantum subsystem. The optimal control signals are designed based on necessary optimality conditions. An illustrative simulation, for population transfer in a 4-level closed quantum system, will be performed to evaluate the applicability of the proposed approach.

The main contributions of this paper are as follows. First, a switching optimal adaptive controller is designed for trajectory tracking of the finite closed quantum dynamic systems. Second, we establish 2 theorems to guarantee the stability of quantum control system and provide the necessary optimality conditions for the switching optimal adaptive controller. Consequently, the proposed controller optimally manages the singularities in ATT control in quantum systems with least