

# Analysis of reciprocating self-excited induction generator using harmonic balance finite element method

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**Abstract** This paper introduces a general method for analysis of a reciprocating self-excited linear induction generator based on the harmonic balance finite elements method (HBFEM). A system of equations is obtained by combining diffusion equation and circuit equations. This system calculates the minimum required capacitance to produce the self-excited mode. It also gives the steady-state response without analysis of the transient mode of operation. The calculated value of the minimum required capacitance for self-excitation process is used as an initial value, and the capacitance is gradually increased using an iterative procedure, therefore the required capacitance is obtained for the proposed voltage at a fixed load. To verify the accuracy of the recommended method, the results based on the proposed method and the time stepping finite elements method (TSFEM) are compared.

**Keywords:** Harmonic balance finite element, Self-excited induction generator, Excitation capacitor.

## 1. Introduction

Induction generators are inexpensive and with no separate excitation. They can operate over a long period with no a serious maintenance. Therefore, self-excited induction generators (SEIG) are important from electrical power generation point of view in the area far from the power system grids and where non-conventional energies such as wind energy are available. However, SEIGs have own disadvantages including large dependency on the output voltage, the generator speed, load and the stator terminal capacitance requirement, which requires to account for the speed and load variations. Two objects are followed in this paper. The first is finding the minimum required capacitance to build up the self-excitation and the second is obtaining generator steady-state voltage and current waveforms. The models that are used for analysis of the SEIG are classified into two major groups. In the first group, phase equivalent circuit is utilized; in this case nodal-admittance or loop-impedance method is used to establish the relationships between the machine-related parameters such as load, speed and capacitance [1]. The second group uses dq model; other equations expressing dependency of the steady-state parameters of the machine are obtained using the harmonic balance method [2, 3]. In the above-mentioned methods, an attempt has been made to solve a non-linear equation by an iterative procedure, where experimental data for magnetization inductance (versus magnetization current) are available.

This paper uses finite elements method for modeling induction generators. Since saturation is unavoidable in SEIGs, the third harmonic currents and voltages are appeared. It means that it is not possible to assume sinusoidal waveforms for voltages and currents and

therefore frequency domain finite elements method is not applicable [4]. So, analysis of SEIGs is carried out using time domain finite element method (TSFEM) [5, 6] and this requires a long computation time for simulation of the transient mode up to the steady-state mode.

In this paper a general method of HBFEM [7-9] is recommended for analysis of SEIG; in this manner, SEIG equations are converted to a matrix form static FEM equations by the TSFEM. These equations are used to obtain the minimum required capacitance of the self-excitation process and waveforms of the state variables of steady-state operation of SEIG at a given load. As an example, a reciprocating tubular SEIG, shown in Fig. 1, is considered and its performance is predicted using the HBFEM.

## 2. Harmonic balance finite element formulation

The diffusion equation for two-dimensional formulation of magnetic vector potential  $A_\theta$  in r-z- $\theta$  axisymmetric coordinate is:

$$\frac{\partial^2 A_\theta}{\partial r^2} + (1/r)(\partial A_\theta / \partial r) + \frac{\partial^2 A_\theta}{\partial z^2} - A_\theta / r = \mu \sigma (\partial A_\theta / \partial t) + \mu \sigma V_z (\partial A_\theta / \partial z) - \mu J_\theta \quad (1)$$

a time periodic profile for the reciprocating motion of the translator is considered as follows:

$$V_z(t) = \sum_{n=1}^{N_h} V_{nc} \cos(n \omega t) + V_{ns} \sin(n \omega t), \quad (2)$$

$$\omega = 2\pi f, \quad (f = 34 \text{ Hz}, N_h = 4)$$

The elements of stiffness matrix is obtained by Galerkin approach and the HBFEM formulation as follows: