Fault location on branched networks using mathematical morphology

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Abstract: Here, a new travelling wave-based fault location method is proposed for branched networks. A mathematical morphology-based filter is used to analyse fault-induced transient signals and detects the arrival time of travelling waves. In the proposed algorithm, using the first surge arrival time of travelling waves at each of the travelling wave detectors installed in the network, a set of linear equations is constructed. By solving the equations, the probable fault occurrence points on the main lines and the main line terminations are identified. Finally, two appropriate evaluation criterions are introduced to determine accurate faulted lateral branch or fault location on the main lines. Extensive simulation studies using EMTP and MATLAB are carried out to examine the effectiveness of the proposed method. The obtained results verify the high accuracy, noise immunity, and fault impedance robustness of the proposed method.

1 Introduction

Faults in distribution networks affect power system reliability, security, and quality. Accurate fault location (FL) has a major impact on the service continuity, and also minimises the time needed to restore power and reduces costs. FL in distribution networks is a challenging task for distribution networks with several laterals, non-symmetrical lines, highly unbalanced operation, and time-varying loads. Thus, the FL method for transmission lines cannot be easily used for distribution systems. Various methods have been proposed for FL in distribution systems. Generally, FL methods can be grouped into three main categories: (i) impedance-based methods [1–7]; (ii) travelling wave-based methods [8–16]; and (iii) methods based on neural networks [17, 18]. Impedance-based methods estimate the fault distance by comparing apparent impedance with the pre-known line data. Their application to distribution networks with some laterals, sublaterals, and load taps may result in a decrease in the location accuracy. On the other hand, travelling wave-based FL methods have been widely used in transmission lines. Such methods can be classified as methods based on wave-head identification (single-end [14, 19], two-end [13], and multi-end [16, 20]) and characteristic frequencies [9]. The most commonly used single-end algorithm for transmission line reflectometry. However, reflected waves at discontinuity points in branched distribution networks are hard to analyse because of large numbers of laterals. Some approaches use artificial neural networks (ANNs) for FL in distribution networks [21–25]. Some ANN-based FL methods employ fundamental frequency components [21, 22] and others employ high-frequency components of fault originated waves [23–25]. In these methods, influence of the load condition and fault inception angle maximises complexity of the algorithm. Some of methods identify location of faults by estimating the path characteristic frequency in distribution networks [9, 17, 26]. In these methods, the accuracy of algorithm depends on estimating path characteristic frequency. In addition, these methods cannot be easily used for any distribution network. Recently, a method based on travelling wave has been proposed for FL in distribution networks [27]. However, the main problem in this method is that each branch termination must be equipped with a fault locator. In most of the mentioned methods based on travelling wave and characteristic frequencies, wavelet transform (WT) has been used for extracting and processing transient signals. Although WT is a strong tool for signal processing and has good time–frequency resolution, in some cases, it may be influenced by the disturbance caused by noises. In faulty conditions with low inception angle and high impedance fault, the amplitude of travelling waves becomes too small and it becomes difficult to detect them by WT [28].

In contrast to WT, mathematical morphology (MM) obtained from the set theory and integral geometry [29, 30] has been developed as a significant tool in geometrical description and analyses. MM has been extensively used in image and signal processing. Since the only mathematical calculations in MM are addition and subtraction, therefore, its calculation volume is low and FL based on this method is very fast and accurate. Taking the above problems into account, this paper proposes a new FL method based on travelling wave in distribution networks which employs a limited number of travelling wave detectors (TWDs). An appropriate MM filter is used for processing transient signals and detecting the arrival of times. Opposite to the method in [27], in the proposed method, only the main line terminations of the network are needed to be equipped with TWD. According to the first arrival times at different TWDs installed in the network, a set of linear equations are introduced to determine the location and fault inception time. By solving these linear equations, probable fault occurrence points on the main lines are identified. Finally, two appropriate evaluation criterions (ECs) are introduced to accurately determine FL on the main lines and faulted lateral branch in the distribution network.

2 Mathematical morphology

Accurate operation of travelling wave-based FL requires a robust tool for processing transient signals. MM is a non-linear analysis technique with high accuracy and low computation burden which can be used for extracting the information of high-frequency signals that result from different power system disturbances. It is also focused on the shape of signals in the time domain and needs a shorter data window [31]. In MM, a structural element (SE) is used for extracting the feature of the original signals [32]. The choice of SE is important for processing the signal with MM and depends on the particular application. An SE could have different shapes, some of which are flat, square, curve, triangular, and semi-circular [33]. In the power systems in which signals are one dimensional, the most suitable SE is flat. In this paper, the flat SE \(g(m) = (0, 0, 0)\) was considered.

Dilation and erosion are two basic operations in MM. Based on these two operations, other compound operators such as opening and closing are defined. Assuming that \(f(n)\) is the input signal as defined discrete function at amplitude \(D_j = (0, 1, 2, \ldots, N − 1)\) and...
appropriate morphological filter (MF) is defined as [31]:

\[
\text{MMF}(n) = \begin{cases} 
\text{MF}_d(n) - \text{MF}_e(n), & \text{MF}_d(n) > \text{MF}_d(n-1) \\
\text{MF}_e(n) - \text{MF}_d(n), & \text{MF}_e(n) < \text{MF}_e(n-1) 
\end{cases}
\]

Based on (6), when the dilation signal leads the erosion signal, MMF output has positive polarity; when a dilation signal lags the erosion signal, MMF output has negative polarity. Also, when there is no sudden change in the original signal, none of the erosion and dilation signals have phase differences relative to each other and the MMF output becomes zero. The processing results of signal \( f(n) \) by the MMF are shown in Fig. 1. It is clearly observed that the MMF could accurately detect the signal polarity.

3 Proposed FL algorithm

3.1 Determining probable fault inception time and fault occurrence points on the main lines

Fig. 2 shows part of the distribution network with only one main line between buses \( i \) and \( j \) and \( K \) lateral branch which are connected to the main line at points \( J_1 \) to \( J_K \) and there terminations are labelled \( E_i \) to \( E_j \), respectively. Two TWDs are installed at the buses \( i \) and \( j \). Assume each TWD monitors the three-phase voltage and is equipped with a global positioning system (GPS) and appropriate communication link. The TWDs are able to detect travelling wave and record arrival time. The GPS synchronises the detectors accurately. Assume that a fault occurs at arbitrary position on the main line and lateral branches. If the fault occurs at \( f_i \) on the main line, the first surge arrival times identified by detectors \( D_i \) and \( D_j \) are denoted by \( t_{D_i} \) and \( t_{D_j} \), respectively, and can be described by the following equations:

\[
t_{D_i} = T_{i}^{x_f} + x_f^{i,j}T_{ij}
\]

(7)

\[
t_{D_j} = T_{j}^{x_f} + (1 - x_f^{i,j})T_{ij}
\]

(8)

where \( T_{i}^{x_f} \) is the fault inception time, \( T_{ij} \) the surge travelling time between buses \( i \) and \( j \), and \( x_f^{i,j} \) the fraction of the total length of the path between buses \( i \) and \( j \). Accordingly, \( 0 \leq x_f^{i,j} \leq 1 \). Also, \( T_{ij} \) can be easily obtained from

\[
T_{ij} = \frac{L_{ij}}{v}
\]

(9)

where \( L_{ij} \) is the length of the shortest path between buses \( i \) and \( j \) and \( v \) the speed of travelling wave. By combining (7) and (8):

\[
T_{i}^{x_f} = \frac{t_{D_i} + t_{D_{ij}}}{2}
\]

(10)

\[
x_f^{i,j} = \frac{t_{D_j} + T_{ij}}{2T_{ij}}
\]

(11)

where

\[
t_{D_{ij}} = t_{D_i} - t_{D_j}
\]

(12)

\[
t_{D_{ij}} = t_{D_i} - t_{D_j}
\]

(13)

Therefore, it is clear that the distance to fault from bus \( i \) is determined as

\[
d = x_f^{i,j}L_{ij}
\]

(14)

According to (10) and (11), if fault occurs on the main line between buses \( i \) and \( j \), accurate FL and fault inception time can be determined using the first surge arrival times recorded by the detectors \( D_i \) and \( D_j \). Therefore, (10) and (11) can be used to...
determine probable fault inception time and fault occurrence points on main lines in the distribution network. Suppose that \( N \) detectors are installed in the network. Here, using (10) and (11), a probable fault occurrence point can be determined for each path between the two detectors. Therefore, for a network with \( N \) detectors, the number of probable fault occurrence points on the main lines is denoted by \( C(N, 2) \) and given by

\[
C(N, 2) = \binom{N}{2} = \frac{N!}{2!(N-2)!}
\]  

(15)

It should be noted that (10) and (11) are valid only for the shortest path between the detectors \( D_i \) and \( D_j \). Unlike the transmission networks, in the branched distribution networks, only one path exists between any two detectors and thus, is the shortest path.

### 3.2 EC to determine accurate FL and fault inception time on the main lines

As explained in the previous section, all probable points of fault occurrence and fault inception times on the main lines in the network are determined which are equal to \( C(N, 2) \). In this section, an EC is introduced to determine only one point among a set of probable points of fault occurrence and fault inception times. To better understand this point, refer to Fig. 3 in which the fault is assumed to occur on the branch \( J_m - J_0 \) on the path between the detectors \( D_i \) and \( D_j \). All the detectors sensing the first travelling wave are divided into two groups. The first group contains the detectors whose first arrival surges have traversed node \( J_m \). Also, the second group includes the detectors whose first arrival surges have traversed node \( J_0 \). It is assumed that the number of detectors is \( X \) in the first group and is \( Y \) in the second group. Therefore, for the first group of detectors

\[
\begin{bmatrix}
T_{D_1} - T_{J_mD_i} \\
T_{D_2} - T_{J_mD_i} \\
\vdots \\
T_{D_{x_1}} - T_{J_mD_i}
\end{bmatrix} = \begin{bmatrix}
T_{D_1} \\
T_{D_2} \\
\vdots \\
T_{D_{x_1}}
\end{bmatrix} \left( x^{i/j} \right) Y_{x_1} \quad \forall i \in X
\]  

(16)

where \( T_{D_0} \) is the first surge arrival times identified by the detector \( D_i \) for the branch \( J_m - J_0 \). The time during which the travelling wave propagates through the shortest path between node \( J_m \) and bus \( i \) on which the detector \( D_i \) is installed, and \( T_{J_mD_i} \) the surge travelling time between nodes \( J_m \) and \( J_i \). Also, \( x, j \) is the fraction of the total length branch \( J_m - J_0 \). Obviously

\[
\chi_{i,j}^{i/j} = \frac{x^{i/j}T_{D_j} - T_{J_mD_i}}{T_{J_mD_i}}
\]  

(17)

Also, for the second group of detectors

\[
\begin{bmatrix}
t_{D_{x_1}+1} - T_{J_mD_{x_1}+1} \\
t_{D_{x_2}+1} - T_{J_mD_{x_2}+2} \\
\vdots \\
t_{D_j} - T_{J_mD_j} \\
t_{D_N} - T_{J_mD_N}
\end{bmatrix} = \begin{bmatrix}
1 \\
1 \\
\vdots \\
1 \\
1
\end{bmatrix} \left( x^{i/j} \right) Y_{x_1} \quad \forall i \in Y
\]  

(18)

The value of \( EC \) is theoretically equal to zero, and in practice would take a very small value for the fault occurrence point \( x^{i/j}, Y_{x_1} \). Therefore, EC is calculated for all the probable fault points and the point with minimum EC is selected.

### 3.3 Determining the faulted lateral branch

In this section, EC is introduced to determine faulted lateral branch in the distribution network. According to Fig. 2, if the fault occurs at \( f_1 \) on the lateral branch \( J_K - E_K \), the first surge arrival times identified by the detectors \( D_i \) and \( D_j \) can be derived as:

\[
t_{D_i} = T_{D_0D_i} + T_{F_1D_i} + T_{J_fD_i}
\]  

(20)

\[
t_{D_j} = T_{D_0D_j} + T_{F_1D_j} + T_{J_fD_j}
\]  

(21)

where \( T_{J_fD_i} \) and \( T_{J_fD_j} \) are the times a surge needs to travel the direct paths from node \( J_f \) to detectors \( D_i \) and \( D_j \), respectively. In addition, \( T_{F_1D} \) is the time a surge needs to travel the direct paths from the fault point \( f_1 \) to node \( J_K \). By combining (20) and (21)

\[
t_{D_i} - t_{D_j} = T_{J_fD_i} - T_{J_fD_j}
\]  

(22)

To identify the lateral branch \( J_K - E_K \) as the faulted branch, (22) can be used. Therefore, (22) is applied for all lateral branches in the network. Thus, EC for the lateral branch \( J_K - E_K \) as the faulted branch can be defined as

\[
EC(J,E) = \sum_{\forall i,j \in N} \left[ t_{D_i} - t_{D_j} - (T_{J_fD_i} - T_{J_fD_j}) \right] \quad \forall i,j \in N
\]  

(23)

where Path \# \( DD_j \) specifies the path between the detectors \( D_i \) and \( D_j \). The value of \( EC(J,E) \) is theoretically equal to zero, and in practice would take a very small value for the faulted lateral branch \( J_K - E_K \). Therefore, by comparing the obtained ECs from (19) and (23), the minimum EC can be determined that is associated with actual faulted lateral branch or actual FL point on the main lines in