Analysis of conjugate natural convection within a porous square enclosure occupied with micropolar nanofluid using local thermal non-equilibrium model

S.A.M. Mehrayan⁎,⁎⁎, Mohsen Izadia, Mikhail A. Sheremetc,d

⁎ Young Researchers and Elite Club, Yasooj Branch, Islamic Azad University, Yasooj, Iran
⁎⁎ Laboratory on Convective Heat and Mass Transfer, Tomsk State University, 634050 Tomsk, Russia
a Department of Nuclear and Thermal Power Plants, Tomsk Polytechnic University, 634050 Tomsk, Russia
b Mechanical Engineering Department, Faculty of Engineering, Lorestan University, Khorramabad, Iran
c Young Researchers and Elite Club, Yasooj Branch, Islamic Azad University, Yasooj, Iran

KEYWORDS
Conjugate natural convection
Porou s square cavity
Micropolar nanofluid
Local thermal non-equilibrium model
Numerical results

ARTICLE INFO
Article history:
Received 17 September 2017
Received in revised form 16 November 2017
Accepted 30 November 2017
Available online 5 December 2017

ABSTRACT
This work aims to study the conjugate natural convection of micropolar nanofluid within a porous enclosure considering local thermal non-equilibrium model. The Galerkin finite element method is employed to solve the coupled and non-linear equations. The governing parameters are Darcy–Rayleigh number, prandtl number, vortex viscosity parameter, ratio of wall thermal conductivity to that of the base fluid, interface parameter (Kr, H) in conditions that declines with thickness of the solid wall and porosity. The Nusselt numbers for both phases in the porous medium significantly decline as thickness of the solid wall rises, with the exception of d = 0.35. Also, it can be concluded as the porosity parameter increases for the passing flow, the nanofluid flow is governed by the classic Navier-Stokes equations.

© 2017 Elsevier B.V. All rights reserved.

1. Introduction

The micropolar fluid [1,2] is an improvement of Navier–Stokes theory, which transform the classical continuum fluid mechanics to microcontinuum by taking into account the effects of micro-rotation of fluid molecules. This approach represents those rigid-form molecules of micropolar fluids could physically spin independent of main stream and their local velocity. It’s seems that numerous discrepancy of experimental and numerical data arises from such micro structure effects. Therefore, micropolar theory uses from concept of vortices to describe flow characteristics. They represent a key physical mechanism related to suspensions, liquid crystal, biological fluid and nanofluids. Hsu [3] numerically studied natural convection flow of a micropolar fluid in an enclosure. Also, numerical study of natural convection flow in a tilting enclosure filled with micropolar fluids is another work Hsu [4]. Hsu and Hong [5] corroborate the research done by Aydin and Pop [6] that micropolar fluids presented lower heat transfer values than those of the Newtonian fluids. Also, Aydin and Pop [7] analyzed the steady laminar natural convective flow and heat transfer of micropolar fluids using a two-dimensional numerical simulation. Because of decreasing overall heat transfer, Zadravec et al. [8] showed that micro-rotation of particles in suspension should not be neglected when computing heat and fluid flow of micropolar fluids.

Recently, many studies have focused on the convection mechanism of micropolar fluids [9,10]. For example, authors in [10] performed an analysis of natural convective flow inside of a cavity filled by a micropolar fluid. They stated that the micro-rotation and fluid velocity increase and decrease respectively while the vortex viscosity parameter increases. Also it was concluded that the form of streamlines is dependent on the value of vortex viscosity parameter. The study on laminar natural convective of micropolar fluid inside a trapezoidal cavity was numerically fulfilled by Gibanov et al. [9]. The authors considered the effects of Rayleigh number, Prandtl number and vortex viscosity parameter. The outcomes showed that an increase in the vortex viscosity parameter and the Prandtl number results in debilitation and intensification of the convective heat transfer and fluid flow, respectively. Saleem et al. [11] performed the study of transient natural convection associated to a micropolar fluid flow in a rectangular enclosure having a heated bottom wall and two cold vertical walls. They showed that the heat transfer rate of non-Newtonian micropolar fluid is less than that of the Newtonian fluid. The more studies on micropolar fluid flow

https://doi.org/10.1016/j.molliq.2017.11.177
0167-7322/© 2017 Elsevier B.V. All rights reserved.
indexes. The periodic natural convection in an enclosure can be seen in works performed by Sheremet et al. [12]. Kolsi have provided insights into the thermal behavior of nano fluids which refer to the class of fluid with suspended nanoparticles made of different materials by chemical and physical processes. This new generation of fluids has been attracted numerous interests in order to manifest their hydrodynamic and thermal performance. Li and Eastman [15], Pak and Cho [16], Xuan and Li [17], Mehryan et al. [18] and Izadi et al. [19], Al-Rashed et al. [20], Kolsi et al. [21], Noghrehabadi et al. [22,23] and Sheikholeslami [24–29] have provided insights into the thermal behavior of nano fluids in confined flows and have confirmed their superior improvement of thermal indexes. The periodic natural convection in an enclosure filled with nanofluids was considered by Ghasemi and Aminossadati [30]. The results indicate that adding of Cu nanoparticles into base fluid enhances the heat transfer especially at low Rayleigh numbers. Using two-phase mixture model, Toosi and Siavashi [31] considered natural convection of a nanofluid inside a square cavity filled by partially porous medium. They showed that optimal volume fraction and porous layer thickness could achieve to maximum Nusselt number for different values of the Rayleigh and Darcy numbers. Yousaf and Usman [32] presented the numerical results of natural convection in a two-dimensional square cavity in the presence of roughness on vertical walls. It was shown when the sinusoidal roughness elements were located on both the hot and cold walls simultaneously the maximum reduction in the average heat transfer was 28%. Sheikholeslami and Sadoughi [33] very recently have simulated the enhancement of CuO-water nanofluid heat transfer with melting surface. More studies on the flow and heat transfer related to the different types of nanofluid under variable conditions can be seen in [34–43].

On the other hand, some researchers have been shown a gap between the numerical and experimental findings of natural convection throughout nanofluid flows. It is possible that disagreement between numerical and experimental results could be due to neglecting the effect of micro-rotation in the Navier–Stokes theory. Therefore, a new model of nanofluids as micropolar fluid has been represented by authors. Bourantas and Loukopoulos [44] investigated the flow of MHD micropolar nanofluid of Al₂O₃-water driven by natural convection inside a tilted square. Results show that the micro-rotation number forcefully affects the flow characteristics and the convection heat transfer inside the tilted enclosure. At higher value of micro-rotation numbers, circulation and convection become stronger. Bourantas and Loukopoulos [45] theoretically modeled the natural convective flow of micropolar nanofluids. They stated that the micro-rotations in general reduce overall heat transfer from the heated wall and should not therefore be neglected. The theoretical model was validated by comparing with available experimental and theoretical data. Briefly, because the micro structure of nanofluid flow could influence the flow and thermal indexes, it’s important to take into account the effect of micro-rotations for modeling of nanofluids.

A porous medium is distinguished by a material consisting of a solid matrix and pore space. Later allows the fluid flow through the solid matrix. Natural convection throughout porous medium has been applied in various problems including post-accident heat removal from pebble-bed nuclear reactors, high-performance building insulation, geothermal energy, multishield structures used in the insulation of nuclear reactors, solar power collector. The Darcy law was still used to express the relationship between the superficial velocities of flow with the pressure

### Nomenclature

- g: gravitational acceleration
- $C_p$: Specific heat (J/kgK)
- l: cavity size
- d: wall thickness
- V: velocity vector
- N: micro-rotation vector
- k: Thermal conductivity
- p: pressure
- Pr: Prandtl number
- Ra: thermal Rayleigh number
- $x, y$: Cartesian coordinates
- u, v: Components of velocity in x and y directions, respectively
- T: temperature
- $h_{inf}$: convection coefficient at the interface of solid and nanofluid phases.

### Greek symbols

- $\gamma$: stream function
- $\kappa$: vortex viscosity
- $\alpha$: thermal diffusivity
- $\beta$: thermal expansion coefficient
- $\theta$: Non-dimensional temperature
- $\mu$: Dynamic viscosity
- $\varphi$: Volume fraction of nanoparticles
- $\nu$: kinematic viscosity
- $\rho$: density
- $\varepsilon$: Porosity

### Subscripts

- S: solid
- bf: Base fluid
- c: Cold
- h: Hot
- nf: nanofluid
- np: nanoparticles
- w: wall
- h: hot
- c: cool

### Superscripts

- $*: \text{Dimensional variables}$

---

**Fig. 1.** Simple schematic view of the problem.
Table 1
Thermal-physical properties of the base fluid, nanoparticles (see [57]) and solid structure of the porous medium.

<table>
<thead>
<tr>
<th>Fluid</th>
<th>c_p (J/kg K)</th>
<th>k (W/m K)</th>
<th>α × 10⁻⁷ (m/s)</th>
<th>β × 10⁻⁵ (K⁻¹)</th>
<th>ρ (kg/m³)</th>
<th>μ × 10⁻⁴ (kg/m s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>4179</td>
<td>0.613</td>
<td>1.47</td>
<td>21</td>
<td>997.1</td>
<td>8.9</td>
</tr>
<tr>
<td>CuO</td>
<td>540</td>
<td>18</td>
<td>50</td>
<td>29</td>
<td>6500</td>
<td>–</td>
</tr>
</tbody>
</table>

Many studies of convection heat transfer inside heated porous enclosures filled by a nanofluid have been considered [46–55]. Using a meshless technique, Bourantas et al. [49] numerically studied the convection of a nanofluid in a porous square cavity. The results confirm as the solid volume fraction increases, the average Nusselt number also enhances at the presence of a porous medium. However, in the present study the fluid and porous medium are everywhere in local thermal non-equilibrium. Natural convection heat transfer in a three-dimensional porous enclosure filled with a nanofluid using the Buongiorno’s model is presented by Sheremet et al. [56]. Last recently, Mehryan et al. [57] numerically done the analysis related to heat transfer of free convection of Al₂O₃-Cu hybrid nanofluid in a porous square cavity. In the study, the experimental values of thermal conductivity and dynamic viscosity of the hybrid nanofluid were used. Sheikholeslami [58] very recently has carried out the natural convection flow through a porous complex enclosure by employing Darcy law.

In many previous researches role of wall thickness in thermal performance of natural convection inside enclosure have been neglected. However in practical problems thermal conditions of all enclosure are affected by their thick walls. Thus many studies have been taken into account effects of presence of bounded walls, leading well-known conjugate cases [48,59–67]. Liaqat and Baytas [68] have numerically studied laminar natural convection flow in a square enclosure considering thick walls. A meaningful change in the buoyant flow characteristics have been clearly observed as compared to conventional non-conjugate problems. Saied [69] numerically investigated conjugate natural heat transfer in a two-dimensional porous enclosure with finite wall thickness. It’s found that increasing the thermal conductivity ratio and decreasing the thickness of the confined walls can enhance the average Nusselt number. Conjugate natural convection in a partitioned differently-heated square enclosure is numerically presented by Khatami et al. [70]. Authors stated that the average Nusselt number decreases with partition thickness.

Generally an overview of literature clearly shows the importance of wall thickness and material at numerical investigation. In brief, the present paper aims to consider the natural convection of the micropolar nanofluid flow in a porous cavity bounded with conductive thick walls using local thermal non-equilibrium condition. To our knowledge, such study has not been published up to now.

2. Basic equations

A simple schematic view of the conjugate free convection problem under study is observed in Fig. 1. Two solid walls which have the finite thickness of δ*, are located between the horizontal bounds in two sides of a square cavity with size L. Indeed, these solid walls play the role of a conductive interface between the hot and cold walls with the temperature of T∞ and Tc, respectively, and the porous medium occupied by micropolar Cu-water nanofluid. The top and bottom bounds have been insulated against the heat transfer. It is worth mentioning that all the walls are impermeable to the base fluid and nanoparticles. Here, the volumetric heat transfer between the hybrid nanofluid and solid phase of the porous medium is finite and non-zero. This specific type of the heat transfer is considered using the local thermal non-equilibrium model. The nanoparticles always remain suspended in the pores of the cavity.

Using the aforementioned assumptions to derive the equations leads to the following formulations:

continuity equation [71]:

\[ \nabla \cdot \mathbf{V}^* = 0 \] (1)

linear momentum equation [72]:

\[ 0 = -\nabla p^* - \frac{\mu_{nf}}{K} \nabla \cdot (\rho \mathbf{V}^* - (\rho \mathbf{V})_T (T^* - T^*_c) \mathbf{g}) + k \nabla \times \mathbf{N}^* \] (2)

angular momentum equation [44]:

\[ \rho_{nf} j (\mathbf{V} \cdot \nabla) \mathbf{N}^* = \left( \frac{\mu_{nf}}{2} \right) \nabla \cdot \mathbf{N}^* + \frac{k}{\varepsilon} \nabla^2 \mathbf{V}^* - 2 n \mathbf{N}^* \] (3)

Table 2
Grid independency test when Ra = 10⁵, H = Kf = K, Re = 10, d = 0.1, ε = 0.6, Da = 10⁻², Δ = 1 and φ = 0.04.

| Grid size | Nuf | Error (%) | Nu₁ | Error (%) | |ϕ|max | Error (%) |
|-----------|-----|-----------|-----|-----------|------|--------|
| 50 × 50   | 10.793 | 0.130 | 2.6064 | 0.008 | 10.613 | 0.009 |
| 100 × 100 | 10.779 | 0.020 | 2.6104 | 0.153 | 10.590 | 0.216 |
| 150 × 150 | 10.777 | 0.009 | 2.6110 | 0.023 | 10.586 | 0.037 |
| 200 × 200 | 10.776 | 0.009 | 2.6111 | 0.008 | 10.585 | 0.009 |

Table 3
Values calculated for average Nusselt number in a porous triangular shaped enclosure occupied with Cu-water nanofluid.

<table>
<thead>
<tr>
<th>Ra</th>
<th>500</th>
<th>1000</th>
<th>500</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>ϕ</td>
<td>0</td>
<td>0.1</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>Present work</td>
<td>9.64</td>
<td>9.42</td>
<td>13.96</td>
<td>12.85</td>
</tr>
</tbody>
</table>

Fig. 2. The average Nusselt number for this study and Aydin and Pop’s work [7].
Energy equation for fluid phase of the porous medium [60]:

\[
V \cdot \nabla T_{nf} = \frac{\varepsilon k_{nf}}{(\rho C_p)_{nf}} \nabla^2 T_{nf} + \frac{h_{nf}(T_s - T_{nf})}{(\rho C_p)_{nf}}
\]  

Energy equation for solid phase of the porous medium [69]:

\[
0 = (1-\varepsilon)k_s \nabla^2 T_s + h_{nsf}(T_{nf} - T_s)
\]  

Energy equation for solid wall [69]:

\[
0 = \nabla^2 T_w
\]

The vector variables have been presented in these equations by bold letters. The boundary conditions subjected in dimensional \( x \)- and \( y \)-coordinates are.

\[
T_w = T_s \text{ on } x = 0 \text{ and } 0 \leq y \leq L
\]

\[
T_w = T_c \text{ on } x = L \text{ and } 0 \leq y \leq L
\]

| Table 4 |
|------------------|------------------|------------------|------------------|
| \(|\psi|_{\text{max}}\) | \(\Omega_w\) | \(\text{Nuss} \) | \(\text{Nunf} \) |
|------------------|------------------|------------------|------------------|
| 3.614 | 4.559 | 0.112 | 0.344 | 0.1 |
| 8.053 | 3.348 | 0.425 | 2.923 | 1 |
| 16.409 | 1.105 | 1.007 | 10.04 | 10 |
| 3.536 | 4.357 | 0.110 | 0.326 | 0.1 | Saeid [69] |
| 7.898 | 3.232 | 0.418 | 2.814 | 1 |
| 16.219 | 1.090 | 1.010 | 9.887 | 10 |

The present results are compared in Table 4. The comparison between the present numerical solution and that of Saeid [69] at \( Ra = 10^3 \), \( d = 0.1 \), \( K_r = H = 1 \).

<table>
<thead>
<tr>
<th>(Ra=10)</th>
<th>(Ra=100)</th>
<th>(Ra=1000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\psi)</td>
<td>(\psi)</td>
<td>(\psi)</td>
</tr>
<tr>
<td>(Q_w)</td>
<td>(Q_w)</td>
<td>(Q_w)</td>
</tr>
<tr>
<td>(\text{Nuss} )</td>
<td>(\text{Nuss} )</td>
<td>(\text{Nuss} )</td>
</tr>
<tr>
<td>(\text{Nunf} )</td>
<td>(\text{Nunf} )</td>
<td>(\text{Nunf} )</td>
</tr>
</tbody>
</table>

\( Ra = 10 \) \( Ra = 100 \) \( Ra = 1000 \)

\( Ra = 10 \) \( Ra = 100 \) \( Ra = 1000 \)

Fig. 3. Dependency of streamlines (a), isotherms of solid phase (b), isotherms of fluid phase (c) and isolines of micro-rotation (d) on \( Ra \) for pure fluid (solid lines) and nanofluid (dashed lines) at \( \varepsilon = 0.5, \text{Da} = 10^{-2}, K_r = H = 10, K_r = 10, \Delta = 1 \).
It is worth mentioning that \( N^* \) of the above equations represents the \( z^* \)-component of micro-rotation vector. Eliminating the pressure terms in momentum equations is possible using cross partial derivations with respect to \( x^* \) and \( y^* \) momentums. Additionally, by using a new dependent variable namely stream function \( u_{nf} = \partial \psi^*/\partial y^*, v_{nf} = -\partial \psi^*/\partial x^* \), the momentum equations are reduced to single equation expressed below:

\[
\mu_{nf} + \frac{\kappa}{K} \left( \frac{\partial^2 \psi^*}{\partial x^* \partial y^*} + \frac{\partial^2 \psi^*}{\partial y^* \partial x^*} \right) = g(\rho o)_{nf} \frac{\partial T^*_{nf}}{\partial x^*} + \kappa \left( \frac{\partial^2 N^*}{\partial x^* \partial y^*} + \frac{\partial^2 N^*}{\partial y^* \partial x^*} \right)
\]  

(8)

Fig. 4. Dependency of streamlines (a), isotherms of solid phase (b), isotherms of fluid phase (c), isolines of micro-rotation (d) on vortex viscosity parameter for pure fluid (solid line) and nanofluid (dashed line) at \( Ra = 10^5, \varepsilon = 0.5, Du = 10^{-2}, K_c = 1, H = R_c = 10.\)
Hence we have then

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -Ra \left( \frac{\mu_f}{\mu_f + \Delta} \right) \frac{\partial T_{nf}}{\partial x} + Da \left( \frac{\Delta}{\mu_f + \Delta} \right)$$

$$\times \left( \frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2} \right)$$

(10)

and

$$\frac{\partial \psi}{\partial y} \frac{\partial N}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial N}{\partial y} = Pr \left( \frac{\mu_f}{\mu_f + \Delta} \right) \left( \frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2} \right) - 2 Pr \Delta \left( \frac{\mu_f}{\mu_f + \Delta} \right) \frac{N}{2}$$

$$+ \frac{\Delta \mu_f}{\mu_f + \Delta} \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right)$$

$$+ \frac{1}{\alpha_f} \left( \frac{\partial T_{nf}}{\partial y} \frac{\partial T_{nf}}{\partial x} - \frac{\partial T_{nf}}{\partial x} \frac{\partial T_{nf}}{\partial y} \right) = \alpha_f \left( \frac{\partial^2 T_{nf}}{\partial x^2} + \frac{\partial^2 T_{nf}}{\partial y^2} \right) + \frac{(\rho C_p)_f}{\alpha_f} H(T_s - T_{nf})$$

(11)

0 = \frac{\partial^2 T_{nf}}{\partial x^2} + K_f H(T_{nf} - T_s)$$

(12)

0 = \frac{\partial^2 T_w}{\partial x^2} + \frac{\partial^2 T_{nf}}{\partial y^2}$$

(13)

The boundary conditions subjected for dimensionless coordinates are:

$$T_w = 1 \text{ on } x = 0 \text{ and } 0 \leq y \leq 1$$

$$T_w = 0 \text{ on } x = 1 \text{ and } 0 \leq y \leq 1$$

(16 – a)

$$\psi = 0, N = 0, \frac{\partial T_{nf}}{\partial y} = \frac{\partial T_{nf}}{\partial x} = 0 \text{ on } y = 0, 1 \text{ and } d \leq x \leq 1$$

$$\frac{\partial T_{nf}}{\partial y} = 0 \text{ on } 0 \leq x \leq d, 1 - d \leq x \leq 1, y = 0, 1$$

(16 – b)

$$\frac{\partial T_{nf}}{\partial x} = R_k k_{nf} \frac{\partial T_{nf}}{\partial x} - K_f \frac{k_{nf}}{k_f} \frac{\partial T_s}{\partial x} \text{ on } x = d, 1 - d \text{ and } 0 \leq y \leq 1$$

(16 – c)

$$Ra, Pr, \Delta, H \text{ and } Kr \text{ of above equations, respectively, are}$$

$$Ra = \frac{g \beta_f (T_s - T_c) \beta_f}{\alpha_f \nu_f \rho_f}, \mu_f = \frac{\mu_f}{\mu_f + \Delta}, Pr = \frac{\nu_f}{\alpha_f}, \Delta = \frac{\kappa}{\mu_f}, H = \frac{h^2}{\epsilon k_{nf}}, Kr = \frac{\epsilon k_{nf}}{(1 - \epsilon) \kappa}$$

(15)

Fig. 5. Variation of Nusselt number of nanofluid phase (a) and solid phase (b) against the vortex viscosity and Rayleigh number.

R_k appeared in the last equations exhibits a fraction of ratio of wall thermal conduction to that of base fluid so that $$R_k = k_w / k_{bf}$$. 
The heat transfer rate through the solid wall and Nusselt numbers are the physical quantities of interest of the present work defined as

$$Q_w = \frac{q_w L}{k_w} = \int_0^1 Q_w dy = -\int_0^1 \left( \frac{\partial T_w}{\partial x} \right)_{x=0} dy$$

$$N_{u_{nf}} = \int_1^0 N_{u_{nf}} dy = -\int_0^1 \left( \frac{\partial T_{nf}}{\partial x} \right)_{x=0} dy$$

$$N_{u_s} = \int_0^1 N_{u_s} dy = -\int_0^1 \left( \frac{\partial T_s}{\partial x} \right)_{x=0} dy$$

The linear combination of $N_{u_{nf}}$ and $N_{u_s}$ with coefficients $R_k^{-1}$ and $K_r^{-1}$ can also calculate the average heat transfer rate through the solid wall as following.

$$Q_w = R_k^{-1} N_{u_{nf}} + R_k^{-1} K_r^{-1} N_{u_s}$$

2.1. The relations defining the thermo-physical properties

The thermophysical properties of the nanofluid appeared in the governing equations can be calculated by using the relations expressed below [72,73]:

$$\mu_{nf} = \frac{\mu_{bf}}{(1-\phi)^2} + \rho_{nf} = (1-\phi)\rho_{bf} + \phi\rho_{np}$$

$$\alpha_{nf} = \frac{\alpha_{knf}}{\rho_{cp}} = \frac{k_{nf}}{\rho_{cp}}$$

$$\rho_{nf} = (1-\phi)(\rho_{bf}) + \phi(\rho_{np})$$

$$k_{nf} = \frac{k_{np} + 2k_{bf} - 2\phi k_{bf} - \phi k_{np}}{(k_{np} + 2k_{bf}) + \phi(k_{np} - k_{np})}$$
Fig. 7. Variation of $N_{u_{1f}}$ (a) and $N_{u_{2}}$ (b) against ratio of wall to base fluid thermal conduction and volume fraction of nanoparticles.

Fig. 8. Dependency of streamlines (a), isotherms of solid phase (b), isotherms of fluid phase (c), isolines of micro-rotation (d) on $K_r$ for pure fluid (solid lines, $\phi = 0.0$) and nanofluid (dashed lines, $\phi = 0.08$) at $Ra = 10^7$, $\varepsilon = 0.5$, $Du = 10^{-2}$, $R_b = H = 10$, $\Delta = 1$. 
The relation of thermal conductivity is utilized for spherical nanoparticles. Also, the thermo-physical properties of water as base fluid and Cu-nanoparticles of the above relations can be seen in Table 1.

3. Numerical method and validation

Since the partial differential Eqs. (10)–(14) are non-linear and coupled, the use of a numerical approach is essential. Hence, the finite element method is utilized to solve these partial equations subjected the boundary conditions (16). In this numerical approach, first, the strong forms of the equations, which have been represented in Eqs. (10)–(14), are rewritten in equivalent forms namely weak forms. The details of the finite element method can be found in [74,75]. To have an accurate solution which is independent of the number of the elements, it needs to do the grid independency test. The grid independency examination is performed based on assessing the variations of $N_{uf}$, $N_{u}$, and $|\psi|_{max}$ with the grid size. From Table 2, the maximum error because of the variations of grid size, belonged to $|\psi|_{max}$, is 0.3% when grid size enhances from $50 \times 50$ to $100 \times 100$. This means that a $50 \times 50$ mesh can give very satisfactory outcomes. However, to ensure the independence of the results on the grid size at all cases, grid $100 \times 100$ is confidently employed to discretize the computational domain.

To evaluate the correctness and accuracy of our results, comparisons have been performed between the outcomes of this work and those obtained by Aydin and Pop [7], Sun and Pop [46], and Saeid [69]. The excellent agreements found between the results of the present work and reported in literature certify the accuracy of present modeling and simulating (see Fig. 2 and Tables 3 and 4).

4. Results and discussion

This section has focused on the influence of the parameters appeared in the equations and boundary conditions such as Darcy–Rayleigh number $Ra = 10–1000$, porosity $\varepsilon = 0.1–0.9$, interface parameter $H = 1–1000$, $K_r = 0.1–10$, volume fraction of the nanofluid $\phi_{nf} = 0–0.08$, vortex viscosity parameter $\Delta = 0–3$, the width of the solid wall $d =$...
Fig. 10. Surface of $N_{udf}$ (a) and $N_u$ (b) against interface parameters of two phases of porous medium ($K_r$ and $H$).

Fig. 11. Dependency of streamlines (a), isotherms of solid phase (b), isotherms of fluid phase (c), isolines of micro-rotation (d) on $\varepsilon$ for pure fluid (solid lines, $\varphi = 0.0$) and nanofluid (dashed lines, $\varphi = 0.08$) at $Ra = 10^7$, $Da = 10^{-2}$, $H = H_0 = 10$, $K_r = \Delta = 1$. 
0.1–0.4 and ratio of wall thermal conductivity to fluid phase of porous medium on the flow and thermal fields as well as the rates of heat transfer through the solid walls, the solid and fluid phases of the porous medium.

As shown in Fig. 3a, the entry of nanoparticles into the base fluid clearly strengthens the streamlines. In fact, the use of nanofluid specifically increases the buoyancy forces relative to pure fluid due to amplifying thermal conductivity. Also, isotherms of nanofluid phase show slight changes in comparison to pure fluid, which horizontally elongates isothermal lines of both phases through enclosure. Otherwise, slight changes are observed in solid wall isotherms using nanofluid. Viewing the changes in micro-rotations suggests that the use of nanofluid increases the strength of micro-rotation. There are enhanced buoyant effects inside the pores which can apply an angular momentum on nanofluid components by adding nanoparticles to pure fluid. Increasing the Rayleigh number enhances the streamlines, due to the growth of buoyancy force acting on the fluid components. On the other hand, the enhancement of buoyancy forces which is arisen from variation of temperature distribution results in a horizontal elongation of streamlines such that denser streamlines appear near the walls. Following such changes, the natural heat transfer is expected to increase throughout the porous cavity.

As the Rayleigh number increases, the dominance of convection over conduction heat transfer is evident in the isothermal lines of both solid matrix and nanofluid trapped inside the pores (second and third rows in the figure). By increasing the Rayleigh number, the isothermal lines gradually deviate from their parallel state with respect to the wall due to the dominance of natural convection. As Ra enhances, the inclined isothermal lines piecemeal originate from down the hot (left wall) and above the cool wall (right wall) where are connected to the region with higher buoyancy effects. These inclined isothermal lines are developing throughout the solid walls by increasing the Rayleigh number. Moreover, the increase in the Rayleigh number leads to escalated power of micro-rotations accompanied by a change in their pattern as

![Fig. 12. Dependency of streamlines (a), isotherms of solid phase (b), isotherms of fluid phase (c), isolines of micro-rotation (d) on d for pure fluid (solid lines, φ = 0.0) and nanofluid (dashed lines, φ = 0.08) at Ra = 10^7, Da = 10^{-2}, \kappa = 0.5, H = R_k = 10, \Delta = 1.](image-url)
shown in the fourth row in the figure. It arises from the fact increased buoyancy forces exert more angular momentum on fluid particles and therefore intensify rotation of those captured by the pores of porous medium.

As Fig. 4a shows, increasing the vortices viscosity number clearly reduces the strength of streamlines. Increasing the vortex viscosity number amplifies the dynamics viscosity and resistance against fluid movement. On the other hand, the general pattern of flow vortices does not change, but the density of the streamlines near the walls decreases as $\Delta$ augments. Therefore it is expected to be declined the transport phenomena by augmenting $\Delta$. A slight reduction in horizontal prolongation could be observed in the isotherms of both phases of the porous medium by increasing the parameter $\Delta$ (second and third rows in the figure). This originated from the fact that using micropolar nanofluid amplifies the resistance against nanofluid movement due to fortifying fluid viscosity. The fourth row indicates the increase in the power of micro-rotations as the flow vortex viscosity increases.

Surface diagram of Nusselt number for both phases of the porous medium with respect to vortex viscosity and Rayleigh number is demonstrated in Fig. 5. Generally, the Nusselt number of both phases increases by increasing the Rayleigh number and decreasing the vortex viscosity. As shown, for small Rayleigh numbers, the Nusselt numbers of both phases are negligibly affected by vortex viscosity. However, as the Rayleigh number rises, the variations in the Nusselt number become completely dependent on the vortex viscosity.

As illustrated in Fig. 6, the power of flow vortices significantly increases as $R_k$ is increased. This behavior suggests that buoyancy effects are greatly augmented due to dominance of wall conductivity over the fluid one. Moreover, the pattern of vortices in the flow changes and undergoes stretching along the horizontal direction. Therefore, the streamlines become more compact at the region near the walls. Indeed, $R_k$ indicates importance ratio of wall to fluid phase thermal property. At a given $K_s$, increasing in $R_k$ results in a reduction of the thermal resistance of the wall and therefore, thermal resistance of the system (set of walls and cavity). As shown in this figure, the growth of $R_k$ distorts the vertical isothermal lines in the walls as one-dimensional heat transfer is overtaken by two-dimensional one as a result of decreasing in thermal resistance. On the other hand, raises of $R_k$ results in increased density of isothermal lines for both phases in the cavity. In addition, more elongated isotherms of fluid phase indicate enhanced natural heat transfer throughout the enclosure. Micro-rotations are augmented by increasing $R_k$. Moreover, increasing $R_k$ stretches the micro-rotations pattern along the horizontal direction and causes them to become more compact in regions closer to isothermal walls.

Fig. 7 illustrates the variation of Nusselt numbers of the nanofluid and solid phases in the porous medium with respect to the nanoparticle volume fraction and ratio of wall to base fluid thermal conduction. Increment of $\varphi$ and $R_k$ augments $Nuf$ except at low value of $R_k$, while $Nuf$ clearly is independent of $\varphi$. For $R_k = 0.1$ and fluid phase, as seen in Fig. 6, isotherms of nanofluid are approximately parallel to isotherms of pure fluid. Therefore, it is expected the independence of $Nuf$ from $\varphi$. In contrast, at high value of $R_k$ ($R_k = 10$), more horizontal elongation could be observed related to the isotherms by adding nanoparticle to pure fluid and hence, it enhances $Nuf$. Increasing of $Nuf$ with respect
to \( R_k \) could be justified by denser isotherms depicted in second row of Fig. 6. There are similar trends for \( N_u \) as a function of \( \phi \) and \( R_k \) which may be vindicated as an analogous manner.

As depicted in Fig. 8, the power of flow vortices is a slightly increased by increasing \( K_r \), while no changes are observed in the pattern of vortices. Increment of \( K_r \) results in dominating of fluid thermal properties over solid matrix, which decreases thermal resistance attributed to natural convection across the entire cavity. The isotherms of the nanofluid phase undergo more elongation as \( K_r \) increases. In addition, the growth of the streamlines changes the isotherms in the solid walls, which consequently transforms conduction heat transfer in the wall to the two-dimensional case.

Moreover, by increasing \( K_r \), drastic changes are observed in the isothermal lines of the solid matrix of the porous medium and the patterns of both phases become similar, which indicates the porous medium is approaching the state of thermal equilibrium. Such condition is acquired as the role of solid matrix phase fades. Ultimately, the slight increase in the power of micro-rotations caused by increasing \( K_r \), which originates from more applied angular momentum on fluid particles.

Streamlines for both nanofluid and pure fluid flows with respect to \( H \) is demonstrated in Fig. 9. As seen for all cases, a growth of \( H \) increases the maximum value of stream function which indicates that the convection mechanism is increased due to the better thermal interaction between fluid and solid matrix phases inside the single pores. The second and third rows in the figure show that the slight changes in the isotherms of the fluid phase while the pattern of isotherms of the solid phase in the porous medium undergoes a significant elongation as \( H \) increases. Hence it’s expected that variation of \( N_u \) would be more perspicuous rather than the \( N_u_{nf} \) as \( H \) augments. These changes are such that the compactness of the isothermal lines is increased near the solid wall. There is no noticeable alteration at distribution of wall temperature. As shown in the fourth row of the figure, the micro-rotations are slightly increased with \( H \), which originate from further angular momentum applied on fluid particles due to reducing in thermal resistance at interface of fluid-matrix.

Fig. 10 demonstrates the diagram of Nusselt numbers of the nanofluid and solid phases in the porous medium with respect to the two variables \( H \) and \( K_r \). The Nusselt numbers of the two phases increase with \( K_r \). As shown, the contour lines of the Nusselt number for the nanofluid phase are approximately parallel to the axis \( H \) and, therefore, experience slight changes as \( H \) increases. On the other hand, the Nusselt number of the solid phase is considerably increased by increasing \( H \).

Fig. 11 shows the effect of porosity on the streamlines, isotherms of two phases and micro-rotations. Flow with respect to \( \varepsilon \) is demonstrated in the first row of Fig. 11. As shown, increasing the porosity parameter leads to decreased pressure drop and increased power of vortices. In addition, another factor contributing to the increase in the power of vortices is reduction of the resistance against inertia forces caused by the solid matrix. Moreover, transformation of isotherms of the solid wall from vertical to inclined state due to two-dimensional heat transfer is also caused by the increase in the power of vortices. The horizontal elongation of isotherms of the nanofluid phase slightly reduces. However, increasing \( \varepsilon \) changes in the isothermal lines of the solid phase and causes further compactness of isothermal lines near the wall, which may be followed by an increase in the Nusselt number of the solid phase in this region. As the porosity parameter and subsequently the

\[ \text{Fig. 14. Surface variation of } N_u_{nf} \ (a) \text{ and } N_u \ (b) \text{ with Darcy number and vortex viscosity parameter.} \]
porosity dimensions increase for the passing flow, the power of micro-rotations is decreased and, in fact, the fluid is governed by the classic Navier-Stokes equations as pointed out by Piętal [53].

The first row in Fig. 12 evidently shows that the power of vortices declines by increasing the thickness of the solid wall due to amplifying of wall stress to fluid volume ratio acting on fluid particles where connected to solid wall. This dissipative stress stands out against buoyancy effects and also reduces fluid movement. It’s observed a clear reduction in horizontal elongation at the isotherms of the two phases as the wall thickness increases. The change in the pattern of isotherms originates from the predominance of conduction heat transfer in the system due to the increase in the thickness of the solid wall and the decrement in the power of vortices. As concluded from the difference between the patterns of isothermal lines of the two phases in the porous medium, investigating the non-equilibrium conditions is of great significance. The fourth row in the figure indicates that the power of micro-rotations decreases as the wall thickness increases. In fact, this decrease can be attributed to the reduction of the flow strength as a result of increasing of the wall thickness. When the flow strength reduces, the effect coefficient of the convection term in angular momentum equation decrease, which can consequently, decreases the power of micro-rotations.

Surface diagram of Nusselt number for the two phases of the porous medium with respect to the porosity parameter and wall thickness is shown in Fig. 13. Generally, $Nu_{nf}$ is decreased and $Nu_s$ is increased with the growth of the porosity parameter. Nevertheless, the changes in $Nu_{nf}$ and $Nu_s$ are negligible at large and small thickness values, respectively. On the other hand, the Nusselt numbers for both phases in the porous medium significantly reduce as $d$ rises, with the exception of $d = 0.35$, at about which $Nu_{nf}$ slightly raises probably due to the drastic predominance of conduction.

According to Fig. 14, the Nusselt numbers of both phases in the porous medium generally decrease as $\Delta$ and $Da$ increase. In the case of non-micropolar fluids, changes in $Nu_{nf}$ and $Nu_s$ with respect to the Darcy number are so small that they can be considered independent of $Da$.

Diagram of $Q_w$ with respect to different parameters is shown in Fig. 15. For ordinary fluids ($\Delta = 0$), the ratio of convection heat transfer in the porous cavity to conduction heat transfer in the wall is independent of the Darcy number, while a declining trend is observed for the heat transfer rate with respect to $Da$ as vortex viscosity increases. This means that the growth in the ratio of the convective heat transfer in the cavity to the conduction heat transfer in wall is decreased. By increasing $\Delta$, the parameter $Q_w$ reduces, the amount of which is higher at larger Darcy numbers. In fact, as $\Delta$ rises, the power of vortices in the streamlines diminishes; this in turn decreases the ratio of convective heat transfer in the porous cavity to that of conduction in the wall. As shown in Fig. 15b, the increase in $Q_w$ due to an increasing in $Ra$ is greater in ordinary fluids compared to micropolar types. Moreover,
although $Q_m$ is independent of the vortex viscosity parameter at low Rayleigh numbers, the parameters $Q_m$ and $\Delta$ become dependent as Rayleigh number increases. As shown in the diagram for Fig. 15c, $Q_m$ decreases as the thickness parameter and porosity parameter increase. The dependence between $Q_m$ and $\Delta$ becomes more significant at higher values of $R_h$ (Fig. 15d).

5. Conclusions

Conjugated natural convection-conduction heat transfer of a thick porous enclosure filled by a micropolar nanofluid has been analyzed. A comprehensive study related to flow, porous medium and micropolar nanofluid variables as well as characteristics of enclosure wall have been numerically performed using finite element technique. The pertinent parameters in the following ranges: Darcy–Rayleigh number $Ra = 10–1000$, porosity $\varepsilon = 0.1–0.9$, interface parameter $H = 1–1000$, $K_e = 0.1–10$, volume fraction of the nanofluid $\phi_{nf} = 0–0.8$, vortex viscosity parameter $\Delta = 0–3$, the width of the solid wall $d = 0.1–0.4$ and ratio of wall thermal conductivity to fluid phase of porous medium on the flow and thermal fields.

The streamlines, isotherms of solid and fluid phase as well as isomicro-rotations in a wide range of above mentioned parameters have been numerically acquired. Major points could be concluded as follows:

- A slight reduction in horizontal prolongation occurs in the isotherms of both phases of the porous medium by increasing the parameter $\Delta$ due to amplifying fluid viscosity.
- The Nussel number become completely dependent on and free of the vortex viscosity at higher and lower value of the Rayleigh number, respectively.
- Microrotations are augmented and horizontally elongated by enhancing $R_h$.
- Increase of $\phi$ and $R_h$ increases $Nu_{nf}$ except at low value of $R_h$, while $Nu_{nf}$ clearly is independent of $\varepsilon$.
- The slight increase in the power of microrotations caused by increasing $K_e$, which originates from more applied angular momentum on fluid particles.
- The microrotations are slightly increased with $H$, which originate from further angular momentum applied on fluid particles.
- The nanofluid flow is governed by the classical Navier-Stokes equations due to decreasing the power of microrotations as the porosity parameter increase.
- Wall stress to fluid volume ratio acting on fluid particles where connected to solid wall is augments by increasing the thickness of the solid wall.
- In the case of non-micropolar fluids, changes in $Nu_{nf}$ and $Nu_{fl}$ with respect to the Darcy number are so small that they can be considered independent of $Da$.

References

Q. Sun, I. Pop, Free convection in a triangle cavity


