SOME NEW INEQUALITIES INVOLVING HEINZ OPERATOR MEANS

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Abstract. We give some new refinements of Heinz inequality and an improvement of the reverse Young’s inequality for scalars and we use them to establish new inequalities for operators and the Hilbert -Schmidt norm of matrices. We give a uniformly and abbreviated form of the inequalities presented by Kittaneh and Mansarah, and the inequalities presented by Kai and we obtain some of their operator and matrix versions.

1. Introduction

The classical Young inequality says that if \( a, b \geq 0 \) and \( \nu \in [0, 1] \), then
\[
a^\nu b^{1-\nu} \leq \nu a + (1-\nu)b \tag{1.1}
\]
with equality if and only if \( a = b \). Young’s inequality for scalars is not only interesting in itself but also very useful. If \( \nu = \frac{1}{2} \), by (1.1), we obtain the arithmetic-geometric mean inequality
\[
2\sqrt{ab} \leq a + b \tag{1.2}
\]

Kittaneh and Mansarah \cite{4, 5} obtained a refinement of Young’s inequality and its reverse as follows:
\[
a^\nu b^{1-\nu} + r_0 \left( \sqrt{a} - \sqrt{b} \right)^2 \leq \nu a + (1-\nu)b, \tag{1.3}
\]
where \( r_0 = \min\{\nu, 1-\nu\} \).

\[
\nu a + (1-\nu)b \leq a^\nu b^{1-\nu} + R_0 \left( \sqrt{a} - \sqrt{b} \right)^2 \tag{1.4}
\]
where \( R_0 = \min\{\nu, 1-\nu\} \).

Zhao and Wu in \cite{10} obtained refinements of (1.3) and (1.4):

If \( 0 < \nu \leq \frac{1}{2} \), then
\[
a^{1-\nu} b^{\nu} + \nu(\sqrt{a} - \sqrt{b})^2 + r_0(\sqrt{ab} - \sqrt{a})^2 \leq (1-\nu)a + \nu b
\leq a^{1-\nu} b^{\nu} + (1-\nu)(\sqrt{a} - \sqrt{b})^2 - r_0(\sqrt{ab} - \sqrt{b})^2, \tag{1.5}
\]

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