Theory of nonlinear s-polarized plasmon–polariton and phonon–polariton modes in dielectric superlattices

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Abstract

The propagation of nonlinear s-polarized polariton waves (TE modes) in an infinitely extended superlattice is considered. The periodic system is composed of two different components where the layers are arranged in an alternating fashion so that each layer of material 1 is bounded by two layers of material 2 and vice versa. In general, each of the individual layers may be characterized by a Kerr-type nonlinear dielectric function with a frequency-dependent characteristic of either the plasmons in a metal/semiconductor or the optical phonons in an ionic crystal. To investigate the propagation of polariton modes in such a system, a theoretical model is formulated leading to Jacobi elliptic functions for the electric field amplitude across the layers. Subsequently, the application of boundary conditions at the interfaces gives rise to dispersion relations. Numerical examples are given for plasmon–polariton and phonon–polariton modes and a comparison is made with phonon–polariton modes propagating in a three layered system.

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1. Introduction

In recent years, new crystal growth techniques have been developed to investigate the physical properties of specimens consisting of alternating layers that are referred as periodically layered structures or superlattices [1]. Simple superlattices are composed of layers of two different materials having overall quasi-periodicity.

Based on experimental methods, the layers of constituent materials are fabricated in such a way that the thickness $d_1$ of constituent 1 and thickness $d_2$ of constituent 2 have values comparable to atomic spacing up to the order of 100 nm typically.

The propagation of various types of guided and/or surface electromagnetic waves in these layered structures has been the focus of interest both theoretically and experimentally [3–21]. The crystal collective excitations (such as plasmons or optical phonons) that can propagate in these systems are mixed with electromagnetic waves. The resulting coupled modes, which are called plasmon–polaritons or phonon–polaritons, have properties that depend on the physical parameters of the material used and also on the ratio of the thickness of the alternating components [1,2]. The dielectric function of layers with Kerr-type nonlinearity may be considered as being frequency dependent. The nonlinear effects of nonmagnetic homogeneous materials with frequency-dependent dielectric function has already been studied for the propagation of plasmon–polaritons modes in a two interface geometry of a thin film sandwiched by two semi-infinite media [13,17,22]. It was shown that the inclusion of nonlinearity exhibits new branches in the plasmon–polariton dispersion curves, that are absent in the nonlinear frequency-independent approximation [17]. The optically nonlinear behavior of electromagnetic waves in superlattices was studied in such a way that the dielectric
functions of the constituents were considered to be frequency independent [2,23].

This paper is organized as follows: A theoretical framework is presented in Section 2 by giving a general discussion of s-polarized polariton modes in a two-component infinite superlattice, assuming a nonlinear layer with Kerr-type dielectric function sandwiched between two linear layers with frequency-dependent dielectric function. In Section 3, which is devoted to the plasmon–polariton excitations, the linear part of the superlattice unit cell depends on the frequency in such a way that corresponds to electron plasma in a metal or doped semiconductor. In Section 4, where we demonstrate the propagation of phonon–polariton modes, at least one of the layers is assumed to be an ionic crystal supporting optical phonon modes. Section 5 is devoted to conclusions, including the possibility of extending this method to other geometries and other type of dielectric media.

2. Theory

The structures we consider here are characterized as having two alternating media arranged so that each layer of material 2 is bounded by two layers of material 1. Each layer may have different thicknesses and dielectric functions. We seek solutions for the spatial dependence of the electric field amplitude in the z-direction for two distinct regions. The first region is defined by \( nl < z < d_2 + nl \) and is characterized by a Kerr-type nonlinear material \( \varepsilon_2 \). The second region is defined by \( nl - d_1 < z < nl \) and is described by a linear isotropic nonmagnetic material with frequency-dependent dielectric function \( \varepsilon_1(\omega) \). Here, \( n \) is an integer, \( d_1 \) is the linear layer width, \( d_2 \) is the nonlinear layer width and \( l = d_1 + d_2 \) is the periodic length of the two-component (or binary) superlattice. The collective modes are excitations of the whole superlattice structure (Fig. 1).

The dielectric function in the present system is written as follows:

\[
\varepsilon(\omega) = \begin{cases} 
  \varepsilon_{02}(\omega) + \varepsilon_2|E|^2, & j = 2 \quad \text{or} \quad nl < z < d_2 + nl, \\
  \varepsilon_1(\omega), & j = 1 \quad \text{or} \quad nl - d_1 < z < nl,
\end{cases}
\]

where, we assume \( \varepsilon_{02} > \varepsilon_1(\omega) \) to cover both guided waves and surface waves.

The propagation of light in a multilayered system is governed by Maxwell’s equations within each material and we have considered the s-polarized (TE modes) polariton modes in the media [17,22]. The following equations for \( \varepsilon(\omega) \) are obtained for the two different regions:

\[
\left( \frac{\partial^2}{\partial z^2} + \beta^2 \right) E(z) = 0, \quad nl < z < d_2 + nl, \tag{2a}
\]

\[
\left( \frac{\partial^2}{\partial z^2} + \beta^2 \right) E(z) = 0, \quad nl - d_1 < z < nl, \tag{2b}
\]

where \( \beta^2 = \omega^2 / c^2 \left( \varepsilon_{02} - \varepsilon_2 k^2 / \omega^2 + \varepsilon_2 / 2 |E(z)|^2 \right) \) and \( \beta^2 = \omega^2 / c^2 \left( \varepsilon_1(\omega) - \varepsilon_2 k^2 / \omega^2 \right) \).

In general, Eq. (2) may be integrated and expressed in a compact form for both constituents as

\[
\left( \frac{\partial E_j}{\partial z} \right)^2 + \frac{\omega^2}{c^2} g_j \left( |E_j(z)|^2 \right) = C_j, \quad -\infty < z < \infty, \tag{3}
\]

where \( C_j \) represents the integration constant and the coordinate \( z \) (measured perpendicular to the layers) lies within the layer \( j \). The function \( g \) represents a quadratic function of the electric field amplitude and is given by

\[
g_j(z_j, E_j^2) = \left[ \varepsilon_{00}(\omega) - (ck/\omega)^2 + \varepsilon_j E_j^2 \right] E_j^2,
\]

where \( j \) labels the layers.

In these multilayered structures, the excitations are characterized by a Bloch wave number \( Q \) for the direction normal to the interfaces [1,2,23]. We have considered the values of \( Q \) lying within the Brillouin zone associated with the periodic length, where \( 0 \leq Q \leq \pi / l \). Applying Bloch ansatz the dielectric field can be written as

\[
E(z + nl) = \exp (inQl) E(z) \tag{4}
\]

We restrict attention to the edges of the superlattice subbands at \( Ql = 0 \) and \( \pi \) and examine real solutions for \( E(z) \).

The translational symmetry of the system implies that the dispersion relations can be calculated by considering any two adjacent layers only. For simplicity and without the loss of generality, we choose the origin of the axes so that the \( z \) coordinate of the lower and

![Fig. 1. The assumed geometry of a two-component superlattice of infinite extent composed of media 1 and 2 with the unit cell length \( l = d_1 + d_2 \). The dielectric functions of the layers are denoted by \( \varepsilon_1 \) and \( \varepsilon_2 \) with the layer thickness \( d_1 \) and \( d_2 \). The unit cells of the structure are labeled by \( n \).](image-url)
upper boundaries of the first layer are at \([-d_1/2, d_1/2]\) and the boundaries of the second layer are \([d_1/2, d_1/2 + d_2]\).

From the symmetry considerations, it can be shown that the electric field amplitudes at the boundaries are related by [22]

\[
E_{j+1} = \pm E_j, \quad j = 0, 1, 2, \ldots
\]  

(5)

It is convenient to make the following choices: \(E_0 = E(-d_1/2), \ E_1 = E(d_1/2)\) and \(E_2 = E(d_2 + d_1/2)\) as shown in Fig. 1.

The problem of a nonlinear layer bounded symmetrically by semi-infinite linear layers has been treated in Ref. [17], where \(C_1(= C_1) = 0\) and \(C_2 \neq 0\). In the superlattice structure studied here, the layers labeled \(j = 1\) and \(3\) have finite width, therefore we have \(C_1(= C_3) \neq 0\) and \(C_2 \neq 0\).

In the following sections, we consider the propagation of both plasmon–polariton and phonon–polariton excitations in a superlattice structure with a linear/nonlinear unit cell.

### 3. Plasmon–polariton excitations

To investigate the propagation of plasmon–polariton excitations, the same approach developed for a single finite layer is used [17]. The following differential equation for the nonlinear layer is then obtained:

\[
\left( \frac{\partial E_z}{\partial z} \right)^2 + \frac{\omega^2}{c^2} g_2 (E_z^2) = C_2.
\]  

(6a)

For the linear layer,

\[
\left( \frac{\partial E_1}{\partial z} \right)^2 + x^2 E_1^2 = C_1.
\]  

(6b)

From Eq. (6) it can be shown that the integration constants \(C_1\) and \(C_2\) are related by

\[
C_2 = C_1 + \frac{\omega^2}{c^2} \left[ (e_2 - e_1 + p_2) \right] E_1^2,
\]  

(7)

where \(p_2\) represents the nonlinearity coefficient.

To determine the electric field amplitude \(E(z)\) in layer 1, Eq. (6b) can be used leading to two different cases depending on the signs of \(C_1\) and \(x^2 = \omega^2/c^2 (e_1 - n^2)\).

The physical solutions are then characterized by

(a) \(C_1 > 0, \ x^2 > 0\) for guided waves,
(b) \(C_1 < 0, \ \delta^2 = \omega^2/c^2 (n^2 - e_1) > 0\) for surface waves,

where, \(n = ck/\omega\) and is referred to as an effective refractive index.

In the case (a), the field amplitude in the linear layer varies as

\[
\tilde{E}(z) = \frac{\cos(zx)}{\cos(xd_1/2)},
\]  

(8)

where

\[
E(z) = \frac{E(z)}{E_0}, \quad C_1 = \frac{x^2 E_0^2}{\cos^2(xd_1/2)},
\]

(9)

\[
C_2 = \frac{\omega^2}{c^2} \left[ e_2 - e_1 + p_2 + \frac{e_1 - n^2}{\cos^2(xd_1/2)} \right],
\]

and in the case (b), we obtain

\[
\tilde{E}(z) = \frac{\cos h(\delta z)}{\cos h(\delta d_1/2)},
\]  

(10)

where

\[
C_1 = -\frac{\delta^2 E_0^2}{\cos h^2(\delta d_1/2)}, \quad C_2 = \frac{\omega^2}{c^2} \left[ e_2 - e_1 + p_2 + \frac{n^2 - e_1}{\cos h^2(\delta d_1/2)} \right].
\]  

(11)

In the nonlinear medium, for real-type solutions of \(E(z)\), Eq. (6a) can be rewritten as an expression for \(\partial E/\partial z\) and then integrated with respect to \(z\). Following the procedure described in Ref. [17], different cases may arise for the electric amplitude depending on the sign and magnitude of the constant \(C_2\), leading to various possible types of Jacobian elliptic functions [24].

The continuity of \(\partial E/\partial z\) at the boundaries leading to a careful determination of the sign of the integral expression for \(\partial E/\partial z\). Taking this into account, the expression for the field amplitude in layer \(j = 2\) is obtained as follows:

\[
E(z) = \tilde{b}_2 \text{cn}(q_2z + J K_2|m_2),
\]

(12)

where, \(K_2 = \text{cn}^{-1}(0|m_2)\), is one quarter of the period of the elliptic function \(\text{cn}(x|m)\) and the integer \(J\) takes four values 0, 1, 2, 3 for a complete description of the solutions. The argument \(m_2\) is an auxiliary parameter controlling the period and the shape of the elliptic function of interest. The parameter \(q_2\) is an effective wave vector for the \(\text{cn}(x|m)\) function. The quantities \(b_2, q_2\) and \(m_2\) depend on the parameters of the layered structure and are defined in the same manner as in Refs. [17,22].

Applying the boundary conditions at \(z = d_1/2\) to Eqs. (8) and (12), the following dispersion relation for guided waves is obtained:

\[
\tilde{b}_2 \sqrt{D_2} \text{sn} \left( \frac{q_2d_1}{2} + J K_2|m_2\right) - \sqrt{e_1(n^2 - n^2)} \tan \left( \frac{2d_1}{2} \right) = 0,
\]  

(14)

where

\[
D_2^2(\omega) = (n^2 - e_0^2)^2 + 4p_2(e_0^2 - e_1^2) + p_2^2 + (e_1^2 - n^2)/\cos^2(xd_1/2),
\]

\[
q_2(\omega) = (2\pi/\lambda) \sqrt{D_2}, \quad \tilde{b}_2(\omega) = \sqrt{(n^2 - e_0^2 + D_2)/2p_2},
\]

\[
m_2(\omega) = \left( p_2^2 b_2^2 / D_2 \right),
\]

\[
K_2(\omega) = \text{cn}^{-1} (0|m_2), \quad \omega(\omega) = (2\pi/\lambda) \sqrt{e_1(n^2 - n^2)}.
\]  

(15)
For surface waves, from Eqs. (9) and (12) the following dispersion relation are derived:
\[ \delta_2 \sqrt{D_2} \left( \frac{q_2 d_1}{2} + JK_2 m_2 \right) - \sqrt{n^2 - \varepsilon_1(\omega)} \tan \left( \frac{\delta d_1}{2} \right) = 0, \]
where
\[ D_2 = (n^2 - \varepsilon_0(0))^2 + 4p_2(\varepsilon_0(0) - \varepsilon_1(\omega)), \]
\[ + p_2 + (n^2 - \varepsilon_1(\omega))/\cosh^2(\delta d_1/2), \]
\[ \delta = (2\pi/\lambda) \sqrt{n^2 - \varepsilon_1(\omega)} \ p_2 = \frac{1}{2} E_1. \]  

(16)

To make a comparison between frequency-dependent and independent dielectric functions, Eqs. (14) and (16) should be solved numerically. To do so, it is convenient to define the dimensionless parameters: \( \Omega = \omega/\omega_{p1}, \) \( \kappa = \kappa/\omega_{p1}, \) \( R = 2\pi(d/\lambda_{p1}) \) and \( \lambda_{p1} = 2\pi c/\omega_{p1}, \) where \( \omega_{p1} \) represents layer 1 plasma frequency. The necessary condition \( \varepsilon_1(\omega) - \varepsilon_0(\omega) > \varepsilon_1(\omega) \) implies that the physical solutions correspond to the region \( \Omega < \Omega_c \) for surface waves and \( \Omega < \Omega_c \) for guided waves. For the special condition \( \varepsilon_0(0) = 1 \) and \( \varepsilon_1(\Omega) = 1 - 1/\Omega^2 \) we obtain
\[ \Omega_c = \sqrt{\kappa^2 + 1}. \]  

(13)

For a doped semiconductor such as InSb, if we choose: \( \omega_{p1}/2\pi \approx 2.3 \times 10^{13} \text{ Hz}, \) \( \lambda_{p1} \approx 13 \mu \text{m}, \) then \( R = 6.28 \) for \( d_1 = \lambda_{p1} \) and \( R = 12.56 \) for \( d_1 = 2\lambda_{p1}. \)

The dispersion curves for plasmon–polariton modes \( \Omega \) vs. \( \kappa \) are plotted in Fig. 2 and \( \Omega \) vs. nonlinearity (in terms of \( \sqrt{p_2} \)) in Fig. 3, for surface waves.

In Fig. 2(a), the dielectric functions of both layers are assumed to be frequency independent with \( \varepsilon_{01} = 2.23, \varepsilon_{02} = 2.45, R = 6.28, p_2 = 0.05 \) and \( j = 0. \)

In Fig. 2(b), the dielectric functions of the both layers are assumed to be frequency dependent so that \( \varepsilon_1(\Omega) = 1 - 1/\Omega^2 \) with \( \Omega = \omega/\omega_{p1} \) and \( \varepsilon_{01}(\Omega) = 1 - 1/\Omega^2 \) with \( \Omega = \omega/\omega_{p2}, \) where \( \omega_{p1} \) and \( \omega_{p2} \) represent the plasma frequencies of layers 1 and 2, respectively. For this case, the condition \( \varepsilon_{02}(\omega) > \varepsilon_1(\omega) \) implies that \( \omega_{p1} > \omega_{p2}, \) and to perform the calculations it is convenient to express \( \varepsilon_{02} \) in terms of \( \varepsilon_1 \) such that \( \varepsilon_{02}(\Omega) = 1 + (1 - \varepsilon_1(\Omega)), \) where \( \omega_{p2} = t\omega_{p1} \) and \( 0 < t < 1. \) In plotting Fig. 2(b) the parameters used were: \( R = 6.28, p_2 = 0.05, j = 0 \) and \( t = 0.8. \) In Fig. 2(c), \( \Omega \) is plotted as a function of \( \kappa \) for guided waves where both layers are assumed to be frequency dependent.

In Fig. 3(a), both layers are assumed to be frequency independent with parameters \( \varepsilon_1 = 2.23, \varepsilon_02 = 2.45, R = 6.28, j = 1 \) and a fixed value of \( \kappa = 0.5. \) In Fig. 3(b), both layers are assumed to be frequency dependent and the parameters \( R = 6.28, \kappa = 0.5, j = 1 \) were used. Finally, Fig. 3(c) represents \( \Omega \) vs. \( \sqrt{p_2} \) for guided waves, where both layers are assumed to be frequency dependent.

To investigate the behavior of the electric field amplitude, Eqs. (8) and (12) can be used for guided waves and Eqs. (10) and (12) are used for surface
waves. The parameters that are used for determining the field amplitude in linear and nonlinear layers are those that satisfy the dispersion relations, Eqs. (14) and (16).

In the case of guided waves, the variation of the field amplitude $E(z)$ vs. $\tau = z/\lambda$ has been plotted for two different modes in Fig. 4. In Fig. 4(a), $\varepsilon_1 = 2.92, \varepsilon_{02} = 4.0$ and for Fig. 4(b), $\varepsilon_1 = 2.3, \varepsilon_{02} = 2.45$ has been used. In both cases, the nonlinearity coefficient $p_2 = 0.05$ with $j = 0$ and $R = 6.28$ were used. At $z = d_1/2$ with $d_1/2 + d_2$, the functions $\cos(\bar{z})$ and $\text{cn}(\bar{z}m)$ are joint smoothly as expected, since the boundary conditions $\partial E_1/\partial z|_{z=d_1/2} = \partial E_2/\partial z|_{z=d_1/2}$ and $\partial E_1/\partial z|_{z=d_1/2+d_2} = \partial E_2/\partial z|_{z=d_1/2+d_2}$ were employed. The larger part of the amplitude corresponds to $\cos(\bar{z})$ for the interval $[-d_2 - d_1/2, d_2 + d_1/2]$ and the other part represents $\text{cn}(\bar{z}m)$ for the interval $[-d_1/2, d_1/2]$.

4. Phonon–polariton excitations

In the case of phonon–polariton modes, we begin with a three-layer structure consisting of a nonlinear dielectric film of thickness $d$, chosen to occupy the region $|z| \leq d/2$ and characterized by $\varepsilon_2$. The two linear semi-infinite media on each side are characterized by a frequency-dependent dielectric function $\varepsilon_0(\omega)$ in the region $|z| > d/2$. More details can be found in Ref. [17].

It is convenient henceforth to use the vacuum wavelength, $\lambda = 2\pi c/\omega$ as a unit length and introduce a dimensionless variable $\bar{E} = E/E_1$, where $E_1$ is the value of $E$ at the interface $z = -d/2$. Then for the nonlinear film, we obtain

$$
\left( \frac{\lambda}{\partial \bar{E}} \right)^2 = (2\pi)^3 \left[ \frac{c^2}{\omega^2} C_2 - g(|\bar{E}|) \right],
$$

(18)
while for the linear (bounding) media, we may write

$$\left( \frac{\lambda}{c} \frac{\partial E}{\partial x} \right)^2 = (2\pi)^3 \left[ \frac{c^2 k^2}{\omega^2} - \varepsilon_0(\omega) \right] E^2. \quad (19)$$

By analogy with Ref. [17], using Eq. (18) we obtain the following phonon–polariton dispersion relation:

$$\bar{E}_2 \sin \left( \frac{q_2 d_2}{2} - J K_2 m_2 \right) = 1. \quad (20)$$

In the superlattice under consideration, at least one component of the unit cell may be composed of an ionic crystal and the dielectric function of the layer can be chosen to be phonon type, so that we have phonon–polaron–polariton modes may exist. The parameters are for a three-layer structure such as NaCl/Si/NaCl and $\kappa = 0.5$.

$$\bar{E}_2 \sin \left( \frac{q_2 d_2}{2} - J K_2 m_2 \right) = 1. \quad (20)$$

The parameter $\omega_p$ ($l = T, L$ where T stands for transverse optical phonon and L for longitudinal optical phonons) denotes the phonon frequency with the index $j = 1, 2, 3$ representing the different layers.

To simplify the required numerical calculations, the above results are expressed in a more convenient form, by introducing the dimensionless terms $\Omega = \omega_0 / \omega_{0T}$ and $\kappa = k / \omega_{0T}$. We may also define the ratio $\Omega_0 = \omega_{0L} / \omega_{0T} > 1$, which typically lies between 1 and 2 for alkaline halides (e.g. see Ref. [25]). It is also convenient to use a characteristic length parameter (defined in terms of the bounding medium) given by $d_0 = 2\pi c / \omega_{0T}$ and to carry out numerical calculations for the values of the film thickness corresponding to $d = d_0$ and $2d_0$. As an example, in the case of NaBr we have $\omega_{0T} / 2\pi \approx 0.4 \times 10^{12}$ Hz and $\Omega_0 \approx 1.56$, implying that $d_0 = 75 \mu m$. For NaCl, the corresponding values are $\omega_{0T} / 2\pi \approx 4.9 \times 10^{12}$ Hz, $\Omega_0 \approx 1.61$, and $d_0 \approx 60 \mu m$. The condition for bounded solutions as $z \to \pm \infty$, is simply $\kappa^2 / \Omega^2 > \varepsilon_0(\Omega)$, which implies that the physical solutions are restricted to two regions of $\Omega$ characterized by $0 < \Omega < \Omega_c^{(-)}$ and $1 < \Omega < \Omega_c^{(+)},$ where $\varepsilon_0(\Omega) = 1 - 1/\Omega^2$ with

$$\Omega_c^{(\pm)}(\kappa) = \sqrt{\frac{\kappa^2 + \varepsilon_{0\infty} \Omega_0}{2 \varepsilon_{0\infty}}} \pm \sqrt{\left( \frac{\kappa^2 + \varepsilon_{0\infty} \Omega_0}{2 \varepsilon_{0\infty}} \right)^2 - 4 \varepsilon_{0\infty} \kappa^2}. \quad (23)$$

Fig. 5. Plots of $\varepsilon_1(\Omega)$, $\varepsilon_2$ and $(\kappa/\Omega)^2$ vs. $\Omega$ for the special case where $\varepsilon_0 = 10.2$. The shaded regions represent the bands of $\Omega$, where the phonon–polariton modes may exist. The parameters are for a three-layer structure such as NaCl/Si/NaCl and $\kappa = 0.5$.

Fig. 6. Surface phonon–polariton dispersion curves for a three layer system where linear layers 1 and 3 are assumed to be frequency dependent and nonlinear layer 2 is frequency independent. (a) Shows $\Omega$ vs. $\kappa$ for $p_2 = 0.05, R = 6.28$. (b) The same as (a) but plots $\Omega$ vs. nonlinearity (in terms of $\sqrt{\beta_2}$) for a fixed value of $\kappa = 0.5$. The curves labeled by $\Omega_c^{(+)}$ display the bounds of the plasmon–polariton regions.
Fig. 7. Dispersion relation curves for phonon–polariton modes in a superlattice. (a) Surface modes, shows $\Omega$ vs. $\kappa$. (b) Surface modes, represents $\Omega$ vs. $\sqrt{p_2}$. (c) Guided modes, represents $\Omega$ vs. $\kappa$. (d) Guided waves, shows $\Omega$ vs. $\sqrt{p_2}$.

Other conditions may add further constraints on the range of $\Omega$. For example, the assumption that $\varepsilon_2 > \varepsilon_1$ and $p_2 > 0$ ensures that $C_2 > 0$.

As a numerical example, NaCl is chosen to be the bounding medium (with $\varepsilon_{1\infty} = 2.25$, $\omega_{11}/2\pi = 4.9 \times 10^{12}$ Hz, $\omega_{11}/2\pi = 7.9 \times 10^{12}$ Hz and $\Omega_0 = 1.61$) and the film is a medium whose dielectric function has negligible frequency dependence in this range of frequencies. For example, in the case of Si, we have $\varepsilon_{02}(0) = 11.7$. The physical solutions are then constrained to lie within the ranges shown in the shaded regions of Fig. 5, which satisfy the necessary conditions $\varepsilon_{02}(0) > \varepsilon_1(\omega)$ and $\kappa^2/\Omega^2 > \varepsilon_0(\Omega)$, where $\varepsilon_0(\Omega) = \varepsilon_{1\infty}(\Omega_0^2 - \Omega^2)/(1 - \Omega^2)$.

The dispersion relation curves for phonon–polariton modes are plotted in Fig. 6. In the case of surface waves, Fig. 6(a) represents $\Omega$ vs. $\kappa$ assuming $d = \frac{1}{2}d_0$, $p_2 = 0.05$. Also, the solutions for $\Omega$ vs. $\sqrt{p_2}$ with $\kappa = 0.5$ are shown in Fig. 6(b).

For the superlattice case, where the thickness of the unit cell is finite, we have plotted $\Omega$ vs. $\kappa$ in Fig. 7(a), using the parameters $d_1 = d_0/2$, $d_2 = d_0/2$, $p_2 = 0.05$ and $\omega_{1T}$ with fixed $\kappa = 0.5$ in Fig. 7(b), for surface waves. Also, Figs. 7(c) and (d) represent $\Omega$ vs. $\kappa$ and $\Omega$ vs. $\sqrt{p_2}$ for guided waves, respectively, with the same parameters as in Figs. 6(a) and (b).

By comparing Figs. 6 and 7, it can be seen that the influence of the superlattice on the dispersion relations increases the number of branches and shifts them to the left without changing the pattern of the curves.

To investigate the effect of the nonlinearity coefficient on the dispersion relations, we have plotted $\Omega$ vs. $\kappa$ for different values of the nonlinearity coefficient in Fig. 9. As indicated in these figures, increasing the nonlinearity coefficient ($p_2 = 0.001, 0.01$ and $0.1$) and keeping the other parameters fixed, shifts the curves to the left without changing their pattern.

It is then assumed that $\varepsilon_{02}$ (the linear part of the dielectric function of the film) may also be frequency dependent. In this more general case, we express $\varepsilon_1(\omega)$ and $\varepsilon_{02}(\omega)$ in terms of the dimensionless quantities introduced earlier and also define the ratios $s = \omega_{21}/\omega_{11}$ and $t = \omega_{21}/\omega_{1T}$. Depending on the values of the parameters $t$ and $s$ (in particular, whether they are greater or less than 1), different cases may arise. For example, in the case where GaSb is the film material and InAs is the bounding medium, we have

$$\varepsilon_{01}(\Omega) = \varepsilon_{1\infty}\left(\frac{\Omega_0^2 - \Omega^2}{1 - \Omega^2}\right), \quad \varepsilon_{02}(\Omega) = \varepsilon_{2\infty}\left(\frac{s^2\Omega_0^2/\Omega^2}{t - \Omega^2}\right),$$

$t = 0.95$, $s = 0.98$, $\Omega_0 = 1.09$, $\varepsilon_{1\infty} = 12.3$ and $\varepsilon_{2\infty} = 14.4$. 
Here, the physical modes correspond to those bands for which $\Omega$ lies within the intervals defined by

$$0 < \Omega < \min(\Omega_{c}^{(-)}, \Omega_{d}^{(-)}) \quad \text{and} \quad \max(t, 1) < \Omega < \min(\Omega_{c}^{(+)}, \Omega_{d}^{(+)}),$$

(24)

where the characteristic frequencies are defined as follows:

$$\Omega_{c}^{(\pm)}(\kappa) = \sqrt{\frac{\kappa^2 + \varepsilon_{1\infty} \Omega_0^2 \pm \sqrt{(\kappa^2 + \varepsilon_{1\infty} \Omega_0^2)^2 - 4\varepsilon_{1\infty} \kappa^2}}{2\varepsilon_{1\infty}}}$$

(25a)

$$\Omega_{d}^{(\pm)}(\kappa, s, t) = \sqrt{\frac{\kappa^2 + \varepsilon_{2\infty} s^2 \Omega_0^2 \pm \sqrt{(\kappa^2 + \varepsilon_{2\infty} s^2 \Omega_0^2)^2 - 4\varepsilon_{2\infty} \kappa^2}}{2\varepsilon_{2\infty}}}$$

(25b)

For a given three-layer structure such as InAs/GaSb/InAs, the dispersion curves, $\Omega$ vs. $\kappa$ and $\Omega$ vs. $\sqrt{p^2}$, have been plotted in Figs. 8(a) and (b) where $d = d_0$.

From the results presented in this section, we conclude that the phonon–polariton dispersion relation curves show features that are different from those of nonlinear plasmon–polariton modes (e.g. there are typically two bands of frequencies) and are distinctive of the phonon frequencies.

In general, the presence of dielectric nonlinearity introduces many new modes and the nonlinear phonon–polariton modes that can propagate in narrow bands along the planar interfaces between media exhibit interesting properties. In fact, these properties have no counterpart in the case of interfaces between linear media and they offer possibilities for device applications (for example, in optical switching [11,12], and lower-threshold-power devices [28]).

The general case in which both media have nonlinear and frequency-dependent dielectric functions with Kerr-type nonlinearity are under investigation.

5. Conclusions

In this paper, the propagation of surface and guided plasmon–polariton and phonon–polariton modes in a symmetric two-component (or binary) dielectric superlattice were investigated. At least one component of the superlattice unit cell can be a doped semiconductor or a metal having a component with a frequency-dependent dielectric function and the other component has a Kerr-type nonlinear structure. In Section 2, we have shown that the propagation of polariton modes in a symmetric superlattice have Bloch characteristics. The calculations were restricted to the edge sides, namely $0 \leq \Omega \leq \pi$ where $\Omega$ stands for Bloch wave number perpendicular to the interfaces of the layers and $l = d_1 + d_2$ represents the

![Fig. 8. Dispersion curves for surface phonon–polariton modes, where both layers are assumed to be frequency dependent. The bounding curves for the possible physical solutions are shown as $\Omega_c^{(\pm)}$ and $\Omega_d^{(\pm)}$. (a) Shows $\Omega$ vs. $\kappa$ for $p_2 = 0.05$, $R = 12.56$. (b) Represents the plots of $\Omega$ vs. $\sqrt{p^2}$ for a fixed value of $\kappa = 0.5$ and $R = 12.56$.](image)

![Fig. 9. Surface phonon–polariton dispersion curves for different values of the nonlinearity coefficient. Note how increasing the nonlinearity shifts the curves to the left without changing their pattern.](image)
length of the superlattice unit cell. Analytic expressions were derived for the determination of dispersion relations of the Bolch-type modes at the superlattice subband edges. The conditions of the band edges at $Q_l = 0$ and $\pi$ allowed the solution of nonlinear dielectric function using real values of the electric field amplitude.

In Section 3, the plasmon–polariton excitations were considered. In the case of self-focusing nonlinear medium, i.e. $\zeta > 0$, we obtained the implicit dispersion relations for surface and guided plasmon–polariton modes and solved them numerically. Our results show that the number of multiple branches increases more when both layers are frequency dependent as compared to the case of frequency independent layers. The effect of the nonlinearity coefficient on the dispersion curves is in such a way that by increasing nonlinearity and keeping other parameters fixed, the curves are shifted to the left without changing their pattern (Fig. 9).

In Section 4, phonon–polariton excitations were considered. First, for a special case the dispersion curves was plotted in Fig. 6. Then the case of superlattice, where both components of superlattice unit cell were frequency dependent was investigated. The phonon–polariton modes were found to have distinct branches characteristic of optical phonons and showing features that are different from those of plasmon–polariton modes. Finally, the dispersion curves of surface phonon–polariton modes for a given three layered structure such as INAs/GaSb/InAs, where the layers are assumed to be frequency dependent were plotted in Fig. 8, showing an increased the number of branches compares to the previous cases. In general, the presence of nonlinearity introduces many new modes and the nonlinear phonon–polariton modes that can propagate in narrow bands along the planar interfaces between media exhibit interesting properties.

Further studies extending to include complex field values may provide more information for the refractive index, $n = \Omega / \kappa$ inside the subbands (i.e., for any Bloch wave number). Also, these calculations may be extended to study the following:

1. The propagation of plasmon–polariton and phonon–polariton modes in finite superlattices.
2. The effects of a negative nonlinear coefficient ($\zeta < 0$) on these wave modes.
3. The properties of these modes for the cases where the dielectric functions of constituent materials of the superlattice are nonlinear and frequency dependent.

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