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RANKING OF DECISION MAKING UNITS BY UTILIZING DEA AND AHP

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Abstract

Ranking of Decision Making Units (DMUs) by utilizing Data Envelopment Analysis (DEA) and Analytic Hierarchy Process (AHP) is discussed in this paper. We presented a two-stage method for ranking of decision making units. In the first stage, we evaluated each pair of Decision making units separately by using of a proposed DEA method, and in the second stage, the pairwise evaluation matrix generated by results of first stage is utilized for ranking of units via the Analytical Hierarchical Process (AHP).

1. Introduction

Data envelopment analysis (DEA) is a non-parametric method for evaluating relative efficiency in Decision making units (DMUs), which was introduced by Charnes, Cooper and Rhodes in 1978 [1]. While, more than one DMU efficient is evaluated in DEA, then ranking of these DMUs are so important. Up to now many models regarding ranking of these Decision making units in DEA have been presented.

Key Words : *Decision Making Units, Data envelopment analysis, Analytic hierarchy process, Ranking.*

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Analytic hierarchy process (AHP) was introduced by Thomas Saaty in 1994 [3].This method has got many application in economical and social, so it has been developed theoretically and practically so far, and has been utilized in management decision making.

Zilla Sinuany Stern et al. [4] presented a method for ranking of Decision making units by making use of AHP and DEA. In DEA model presented by them, in comparison of each pair of Decision making units, other units are not involved, while in our propose model the evaluation of each pair decision making units is resulted from comparing with the whole decision making units performance.

The paper organized as follows. In Section 2, Data Envelopment Analysis and Analytic Hierarchy Process are briefly discussed. In section 3, our model proposed is presented in section 3. A numerical example is provided and analyzed in section 4. The paper is concluded in section 6.

2. Data Envelopment Analysis and Analytic Hierarchy Process

2.1. Data Envelopment Analysis

Production Technology is a transformation from the inputs x into outputs y. Theoretically production possibility set can be defined as following:

$$T = \{(x, y)| \text{ input } x \text{ can produce the output } y\}.$$

Let n, DMUs are under evaluation with m inputs and s outputs $x_j = (x_{1j}, \cdots, x_{mj})$ and $y_j = (y_{1j}, \cdots, y_{sj})$ are input and output vectors of the decision making unit corresponding with DMU_j. Supposedly here by production possibility set we mean, T set, which was defined by Charnes, Cooper and Rhodes, is as following:

$$T^{CCR} = \left\{ (x, y)| \sum_{j=1}^{n} \lambda_j x_j \leq x, \sum_{j=1}^{n} \lambda_j y_j \geq y, \lambda_j \geq 0 \quad j = 1, \cdots, n \right\}.$$

DEA model for defining the corresponding efficiency with T set for evaluating DMUo as $0 \in \{1, \cdots, n\}$, is as following:

$$
\begin{array}{lll}
Min\ \ \theta & & \\
\text{st.} & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{i0} & i = 1, \cdots, m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{r0} & r = 1, \cdots s \\
& \lambda_j \geq 0 & j = 1 \cdots, n.
\end{array}
$$

The above model is known as CCR model. θ^* the optimum solution of CCR model is related to the efficiency rate of DMUo. $\theta^* = 1$ shows that Decision making unit under evaluation is efficient. Dual of the above model is as following:

$$
\begin{array}{lll}
Max & \sum_{r=1}^{s} u_r y_{r0} & \\
\text{st.} & \sum_{i=1}^{m} v_i x_{i0} = 1 & \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 & j = 1, \cdots, n \\
& u_r \geq 0 & r = 1, \cdots, s \\
& v_i \geq 0 & i = 1, \cdots, m
\end{array}
$$

In this model u_r and v_i are dependent weights to corresponding inputs and outputs with variables of model. They can be interpreted as normalized price which these input and output prices of DMU under evaluation are shown to be the most optimal possible price.

2.2. Analytic Hierarchy Process

The Analytic Hierarchy Process (AHP) is a structured technique for dealing with complex decisions. The AHP helps the decision makers find the one that best suits their needs and their understanding of the problem. It was originally developed by Thomas Saaty [2]. In short, it is a method to derive ratio scales from paired comparisons. The AHP arrange the decision problem as a hierarchical structure containing several levels. The first level defines the main goal of the decision problem and the last (lowest) level usually describes the decision alternatives. The levels in between can contain secondary goals, criteria and subcriteria of the decision problem [Figure 1].

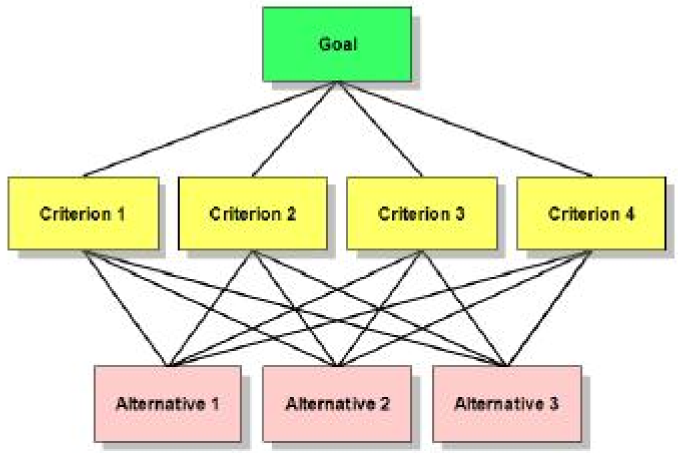

Fig1: A simple AHP hierarchy, with final priorities.

Figure 1 : A simple AHP hierarchy, with final priorities.

Selection of preferences for both criteria and choices is done via pair-wise comparison. After carrying out pair-wise comparisons on criteria with respect to the goal and then choices with respect to each criterion one obtains a matrix representation of the model. The choice is then calculated using linear algebra transformation of the decision matrix. Let us denote the paired comparison matrix $A = \{a_{Ij} | a_{ji} = 1/a_{ij}, a_{ij} > 0, i, j = 1, 2, \cdots, k\}$, where k is the number of elements in the particular comparison set of the lower level. Saaty [3] proposes to use a_{ij} integers in the range 1 through 9 to express preference, where 1 means that the i-th and the j-th element are equally important and 9 means that the i-th element is absolutely more important than the j-th element. The local priorities are derived by solving the following eigenvector problem

$$Aw = \lambda_{Max} w$$

$$1 \cdot w = 1$$

where $.\lambda_{Max}$ is the largest eigenvalue of A and w is the normalized right eigenvector belonging to λ_{Max}. Each component of w introduces the introducer of overall priority corresponding elements.

3. Proposed AHP/DEA Ranking Model

In this section, we have presented a two-stage method for ranking of decision making units. This model is akin to model's Sinuany Stern et al. [4]. Dissimilarly, in the presented DEA model by them in pairwise comparison of Decision Making Units, other decision making units do not bear any role, while in our propose model the evaluation of each and every decision making unit pair is gained by comparing to the function of all the decision making units.

3.1 The First Stage

For each pair unit p and q, the model DEA is considered as following:

$$
\begin{array}{lll}
E_{pq} = & Max & \sum_{r=1}^{s} u_r y_{rp} \\
\text{st.} & & \sum_{i=1}^{m} v_i x_{ip} = 1 \\
 & & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0^j \neq q, j = 1, \cdots, n \\
 & & \sum_{r=1}^{s} u_r y_{rq} - \theta_q^* \sum_{i=1}^{m} v_i x_{iq} = 0 \\
 & & u_r \geq 0 \quad r = 1, \cdots, s \\
 & & v_i \geq 0 \quad i = 1, \cdots, m.
\end{array}
$$

In above model, evaluation of unit p is obtained by the optimal weights of unit q according to proposed model by Oral et al. [5]. th_q^* is efficiency of DMU_q in CCR model.

3.2 The Second Stage

We define the pairwise comparison matrix AHP from the results of DEA for every decision making unit p to q.

$$A = [a_{pq}]_{m \times n}$$

$$a_{pq} = \frac{E_{pq}}{E_{qp}}.$$

Obviously:

$$a_{pq} = \frac{1}{a_{qp}}.$$

Ranking decision making units is based on priorities.

4. Numerical Example

We consider seven decision making units by two inputs for producing a normalized output in the first level. These decision making units are explained in the Table 1.

Table 1 : Properties of the decision making units in the example

DMUs	Input 1	Input 2	Output
A	3	1	1
B	2	2	1
C	2	3	1
E	1	5	1
F	2	5	1
G	3	4	1
H	5	1	1

By comparing the decision making unit pairs, their pairwise comparison matrix is formed. So we have:

$$A = \begin{pmatrix} 1 & 1.11 & 1.51 & 0.66 & 1.85 & 2.16 & 1 \\ 0.90 & 1 & 1.17 & 0.59 & 1.51 & 1.66 & 1.17 \\ 0.66 & 0.59 & 1 & 0.50 & 1.22 & 1.44 & 0.89 \\ 1.51 & 1.17 & 2 & 1 & 2 & 2.56 & 2 \\ 0.54 & 0.66 & 0.82 & 0.50 & 1 & 1.19 & 0.73 \\ 0.46 & 0.60 & 0.69 & 0.39 & 0.84 & 1 & 0.61 \\ 1 & 0.85 & 1.12 & 0.50 & 1.37 & 1.64 & 1 \end{pmatrix}$$

The results are explained in Table 2. As it is seen here, in this ranking efficient units are located in higher level than inefficient units. On there hand, ranking of inefficient units in this method agrees with the rate of their efficiency. Therefore propose model in the essay presents an overall ranking for decision making units.

Table 2 : Results of ranking by our propose model

DMUs	θ^*	w	Ranking
A	1	0.17	2
B	1	0.15	3
C	0.89	0.12	5
D	1	0.23	1
E	0.73	0.10	6
F	0.62	0.09	7
G	1	0.14	4

6. Conclusion

In this paper we presented a two-stage method for ranking of decision making units. In the first stage we have evaluated and compared the decision making units pairs and in the second stage by utilizing Analytic Hierarchy Process, we have presented an overall ranking for efficient decision making units.

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